Rear Wheel Recommendations for Professional Cyclists

or

The Wheel Deal!

Abstract

Professional cyclists are interested in every possible racing advantage, including those gained by choosing the most aerodynamic or power-efficient wheel design. Since a choice exists between solid disk and wire spoked rear wheels, we constructed a mathematical model to augment the decision making process. We considered the physics involved in the problem to derive a second order, nonlinear differential equation of bicycle motion. We developed two different methods to extract mathematical rear wheel recommendations from this physical model; both methods were applied on a variety of racecourse environments. The first method is an algebraic manipulation of the equation of motion, which results in rear wheel recommendations based on average wind velocities, bicycle velocities, and road grades. The second method is a numerical solution to the equation of motion, which results in recommendations based only on average wind road grades and wind velocities. Finally, we compared the recommendations made by each method by performing numerical experiments that tested the validity of both. For over 40 numerical time trials, at least one, and generally both, of the models made a successful rear wheel recommendation.

Contents

1 Introduction

Athletic performance records were once expected to last for significant lengths of time. Amazing physical feats were to live on in the memories of spectators, unmatched for years, or possibly decades. However, as our technological capability and physiological knowledge increase, these records appear to be surpassed every time an athlete steps onto the track, or gears up for a race. In cycling, professional athletes have become interested in the most aerodynamic and power-efficient bicycle configurations. One aspect of the optimum racing bicycle is certainly the wheel design.

Two types of wheels are frequently considered by professional cyclists: those constructed of a solid disk and those constructed using wire spokes. Although track cyclists can use either in the front of the bicycle, the concern of wind effects (e.g. crash anxiety!) favors the choice of wire spokes for the front wheel. Upon viewing racecourse conditions, such as weather, terrain, and wind speed, an educated guess is used to determine the most robust choice for the rear wheel.

Because the rear wheel choice is based on intuition and experience, we sought a mathematical recommendation to augment the decision making process. To this end, the specifications of the rear wheels are listed below. Both wheels are currently in production.

Rear Wheels Compared		
	Conventional 36 spoke	HED disk (lenticular)
Drag Coefficient	0.0491	0.0361
Mass	1.030 kg	1.386 kg
Inertial Moment$_{rotational}$	0.080 kg·m^2	0.095 kg·m^2
Radius	0.337 m	0.337 m

Wheels composed of a solid disk are heavier but more aerodynamic, as is measured by the drag coefficient. A hypothesis, then, regarding wheel performance before approaching the mathematical model is that the solid wheel will outperform the spoked wheel in most measures. To create a mathematically based recommendation, we followed the following five steps:

- Step 1: We considered the physics involved to find a non-linear differential equation for the bicycle motion.
- Step 2: We made physical assumptions to decrease the complexity of the problem.

- Step 3: We manipulated the equation of motion to extract an algebraic relationship between road grade, bicycle velocity, and the wind speed for which the solid wheel required less power input from the cyclist.
- Step 4: We solved the equation of motion numerically, and determined through numerical experimentation the most efficient wheel choice as a function of road grade and wind velocity.
- Step 5: We compared these two methods in order to determine which produced the better recommendations. Among the time trial courses examined was a numerical simulation of the USCF Masters National Road Cycling Championship racecourse to be held this coming July in Spokane, WA.

These steps are now discussed in detail.

2 Analytical Model and Global Assumptions

2.1 Physical Model

To build an informative mathematical model, we first considered the physics involved in the problem. This work produced a nonlinear differential equation on which we based most of our research.

2.2 Force Equations

The following force diagram considers the most significant forces on a bicycle in motion. Each force considered is mathematically interpreted below.

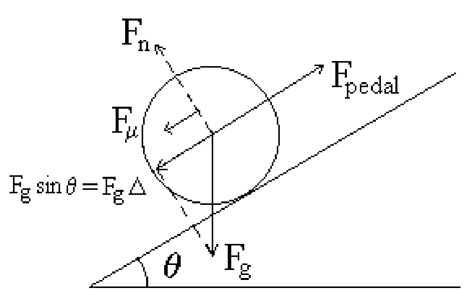

Figure 1: Force Diagram

- Pedaling Force

$$F_{pedal} = \frac{P(t)}{(\frac{dx}{dt})} = \frac{P(t)}{v(t)}$$

where
t = time
$P(t)$ = power
$x(t)$ = distance travelled
$v(t)$ = bicycle velocity

- Air Resistance Force
  The effect of air resistance on a cyclist is proportional to the square of the *relative* velocity through the air. Bicycle motion will not be affected by this force if, for example, a tail wind is blowing at the same speed as the bicycle is travelling.

$$F_a = \frac{1}{2}Ac_w\rho(v(t) + v_w cos(\phi))|v(t) + v_w(t)cos(\phi)|$$

where
A = frontal area of bicycle and rider
c_w = coefficient of air resistance
ρ = air density
v_w = wind speed (directionless)
ϕ = angle of wind direction with respect to forward bicycle motion

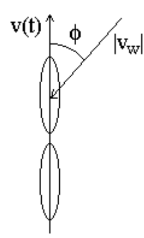

Figure 2: Overhead View of Biker

- Rolling Friction Force
  Because the wheel flattens in contact with the ground, a frictional force is produced that acts against the direction of motion.

$$F_\sigma = C_{rf}Mg$$

where
C_{rf} = coefficient of rolling friction
M = $mass_{bicycleframe} + mass_{cyclist} + mass_{frontwheel} + mass_{rearwheel}$
g = gravity

- Gravitational Force

$$F_g\Delta = Mg\Delta$$

where

$$\Delta = sin(\theta) = \text{road grade}$$

- Drag Force on Front Wheel

$$F_f = \tfrac{\pi}{8}C_{x0fw}d_{fw}^2\rho(v(t) + v_w(t)cos(\phi))^2$$

$$\text{where} \quad \begin{array}{rcl} C_{x0fw} & = & \text{front wheel axial force drag coefficient} \\ d_{fw} & = & \text{front wheel diameter} \end{array}$$

- Drag Force on Rear Wheel

$$F_r = \tfrac{\pi}{8}C_{x0rw}d_{rw}^2\rho(v(t) + v_w(t)cos(\phi))^2(1 - FS)$$

$$\text{where} \quad \begin{array}{rcl} FS & = & \text{effect of bike frame sheltering the rear wheel} \\ C_{x0rw} & = & \text{rear wheel axial force drag coefficient} \\ d_{rw} & = & \text{rear wheel diameter} \end{array}$$

For this study, we held some of the values above constant, as in [1].

A	=	0.5 m^2	I_{fw}	=	0.080 kg·m^2
c_w	=	0.5	ρ	=	1.226 kg/m^3
C_{rf}	=	0.004	$mass_{bicycleframe}$	=	8 kg
C_{x0fw}	=	0.0491	$mass_{cyclist}$	=	65 kg
d_{fw}	=	0.674 m	$mass_{frontwheel}$	=	1.030 kg
g	=	9.8 m/s^2			

2.3 Equation for Bicycle Motion

Using Newton's Second Law, the following equation of motion is produced:

$$\text{Let } F_\mu = F_a + F_\sigma + F_f + F_r$$

$$(M + 4\tfrac{I_{fw}}{d_{fw}^2})\tfrac{d^2x}{dt^2} = \Sigma F = F_{pedal} - F_g\Delta - F_\mu$$

$$\begin{aligned} (M + 4\frac{I_{fw}}{d_{fw}^2} + 4\frac{I_{rw}}{d_{rw}^2})\frac{d^2x}{dt^2} \quad = \quad & \frac{P(t)}{v(t)} - Mg\Delta - \\ & \frac{1}{2}Ac_w\rho(v(t) + v_w cos(\phi))|v(t) + v_w cos(\phi)| - C_{rf}Mg - \\ & \frac{\pi}{8}\rho(v(t) + v_w cos(\phi))^2(C_{x0fw}d_{fw}^2 + C_{x0rw}d_{rw}^2(1 - FS)) \end{aligned} \quad (1)$$

$$\text{where} \quad \begin{array}{rcl} I_{fw} & = & \text{front wheel rotational inertia} \\ I_{rw} & = & \text{rear wheel rotational inertia} \end{array}$$

In order to extract information from this complex equation, making assumptions became necessary. The following is a list of our global assumptions, or assumptions on which we based our entire investigation.

2.4 Global Assumptions / Justifications

1. The cyclist uses a spoked front wheel.
   - This is the most common choice between the two wheels considered.
2. A professional cyclist produces an average power of 350 Watts. (Varying cyclist power inputs are still considered in the model).
   - An average club cyclist consistently produces between 250 and 300 Watts, and the maximum power burst for professional cyclists is approximately 500 Watts. We considered a reasonable average power output in between these parameters [3].
3. The bicycle frame shelters the rear wheel from 25% of all air effects.
   - This assumption was made in an actual physical comparison using a wind tunnel. [1]
4. A wheel's drag dynamics are independent of rotational velocity.
   - In the same physical experiment, an analysis of the results "found that the wheel rotational speed had in fact an almost negligible influence" [1].
5. The only wind component significant in bicycle dynamics is that which is either parallel or anti-parallel to the motion of the bicycle (i.e. $\phi = 0$ or $\phi = \pi$ in figure 2).
   - Experimental work has been done on the moderately significant effect of yaw angle (ϕ) on wheel aerodynamics [1]. However, in order to simplify the problem to manageable proportions, we assumed that adjusting the yaw angle had an equivalent effect on each wheel type and would not influence our results.

With these assumptions, the equation of motion simplifies.

$$(M + 4\tfrac{I_{fw}}{d_{fw}^2})\tfrac{d^2x}{dt^2} = \Sigma F = F_{pedal} - F_g\Delta - F_\mu =$$

$$\begin{aligned}(M + 4\frac{I_{fw}}{d_{fw}^2} + 4\frac{I_{rw}}{d_{rw}^2})\frac{d^2x}{dt^2} &= \frac{P(t)}{v(t)} - Mg\Delta - \\ &\frac{1}{2}Ac_w\rho(v(t)+v_w)|v(t)+v_w| - C_{rf}Mg - \\ &\frac{\pi}{8}\rho(v(t)+v_w)^2(C_{x0fw}d_{fw}^2 - \frac{3}{4}C_{x0rw}d_{rw}^2) \qquad (2)\end{aligned}$$

3 Model Application / Two Strategies

The impetus behind designing a mathematical model for bicycle motion was to develop a systematic recommendation for a choice of rear wheel dependent upon racetrack, road grade, and wind conditions. An initial inspection of the bicycle motion equation incited hope for an analytical solution given proper initial conditions; alas, our hope was for naught (we unable to find one), and we were forced to develop approximating methods upon which to base our recommendations.

We therefore devised two alternative strategies for wheel recommendation: one based on algebraic manipulation of the motion equation, and one based on a numerical solution to the motion equation.

3.1 Algebraic Manipulation for Wheel Recommendation

The goal of this method is to find a regime in the (v_w, Δ) plane for which the solid wheel requires less power to utilize. A systematic analysis of experimentally determined regimes would lead to rear wheel recommendations.

3.1.1 Method Specific Assumptions

- Power requirements are based on local considerations regarding the preservation of velocity and acceleration profiles.
- Local power efficiency translates into global power efficiency for any recommendation using this method.
- In accordance with problem specifics, we assume that a cyclist would lose $8km/h = 2.22m/s$ when traveling $45km/h = 12.5m/s$ up a hill of 5% gradeover 100 meters.

Applying these assumptions, and recalling the equation of motion, we solved for the difference between each wheel's power requirement. Subscripts 'D' correspond to variable values for the equation using a solid 'disk' rear wheel. Subscripts 'W' correspond to values concerning a 'wire' spoke rear wheel.

$$
\begin{aligned}
(P_D - P_W) &= C_1 \frac{dv}{dt} v + C_2 v + C_3 (v^3 + 2v^2 v_w + v v_w^2) \qquad (3) \\
C_1 &= mass_{rearwheel_D} - mass_{rearwheel_W} + \frac{4}{d_{rw}^2}(I_{rw_D} - I_{rw_W}) \approx 0.4881kg \\
C_2 &= (C_{rf} g + \Delta g)(M_D - M_W) \approx 0.013955 + 3.4888\Delta kg \cdot m/s \\
C_3 &= \frac{\pi}{8} \rho d_{rw}^2 (1 - FS)(C_{xorw_D} - C_{xorw_W}) \approx -0.00213243kg/m
\end{aligned}
$$

Manipulating this equation, we were able to extract information about the difference in power required between the two wheel choices. Before considering the usage of this equation, it is necessary to determine the acceleration of the cyclist as a function of the hill grade.

Recalling the third assumption made for this method, one relationship between grade and acceleration can be obtained. The assumption determines that for a constant 5% hill grade, a cyclist will experience an *average acceleration* of -0.2531 m/s^2 (this calculated using common equations in Newtonian physics). Determining other relationships between acceleration and grade is necessary in this method in order to decrease the number of variable parameters in equation (2). Choosing three other relationship "points," we were able to interpolate through the data points to find acceleration as a function of velocity for this method. The three other relationships chosen were the following:

- At $\Delta = -100\%$ grade, the cyclist is "rolling" (falling) backwards at gravitational acceleration, $a = 9.807$ m/s^2 !
- At $\Delta = 100\%$ grade, the cyclist is falling forward at gravitational acceleration, $a = 9.807$ m/s^2.
- At $\Delta = 0\%$ grade, we assume that the cyclist is powering the bicycle at a rate that produces no bicycle acceleration, $a = 0$ m/s^2.

Interpolating through these points for positive hill grades, we obtained a continuous quadratic relationship between grade and acceleration. This relationship, however, is determined by assuming an initial velocity of 12.5 m/s. To find a fully descriptive relationship between grade and acceleration for any initial velocity, we assume that small changes in velocity produce small differences in the quadratic relationship. Finally, we base a mirrored relationship for negative grades upon physical intuition (See figure 3, and further justification below).

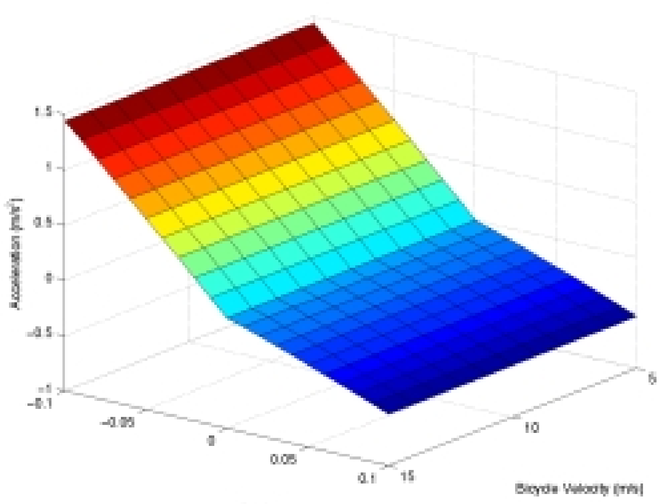

Figure 3: Bike Acceleration versus Bicycle Velocity and Grade

Simply extending the function for negative grades was not consistent with our physical intuition; for short time intervals, we assume that pedaling while biking downhill can increase the acceleration experienced by the cyclist past the acceleration caused by gravitational components. Thus, we propose for this method using the relationship graphically described in figure (3).

After having built the machinery used by this method, we directed our attention to equation (3). Now with acceleration as a function of hill grade, $(P_D - P_W)$ is positive, zero, or negative depending on the bicycle velocity, wind velocity, and road grade. For a choice of these three parameters, the sign of $(P_D - P_W)$ determines which rear wheel requires less power to use. Varying them produced the results discussed in the next section which led to rear wheel choice recommendations.

3.1.2 Method Results and Interpretation

Figures 4-7, included below, graphically describe our results. The dark regions in these figures, such as the hourglass-shaped solid in figure 4, are environmental regimes in which the method finds lower power requirements for *a wire spoked rear wheel.* Thus, dark regions correspond to recommendations for a wire spoked rear wheel. Figures 5 and 6 show horizontal and vertical cross sections, respectively. The area in black is the regime in which the model recommends a spoked wheel.

Since this method bases its recommendations on cyclist velocity, wind velocity, and road grade, there is question regarding its practical value. However, there is a general, slow moving trend through the recommendation regimes. Thus, a realistic use of the method recommendations is to interpret cyclist velocity, wind speed, and road grade as *average* cyclist velocity, *average* wind velocity, and *average* road grade for each racecourse. For average cyclist velocites of 8 m/s and 12.5 m/s, we include Table 1 which, given the road grade, notes the wind velocity regimes for which the model recommends a solid disk rear wheel. The wind velocities in the table are limited to the interval [-15 m/s, 15 m/s] that we explored in our research.

The results obtained by studying this method were aligned with the earlier hypothesis regarding the aerodynamic advantage of a solid disk rear wheel. Considering the volume comparison and directional trend in figure 4, the solid wheel is recommended for a larger realm of environmental conditions. The figures show that for moderate grade and wind velocities, the aerodynamic benefit of choosing a solid wheel outweighs the effects of added mass to the bicycle. The environmental settings for spoked rear wheel recommendations are, however, quite significant. According to this method, cycling in a tailwind or downhill on a significant grade requires less power while using a spoked rear wheel. There is a delicate balance in the equation of motion between gravitational effects and wind velocity effects. Apparently, this method bases its recommendations on this delicate balance, as is seen by both grade and wind direction dependence.

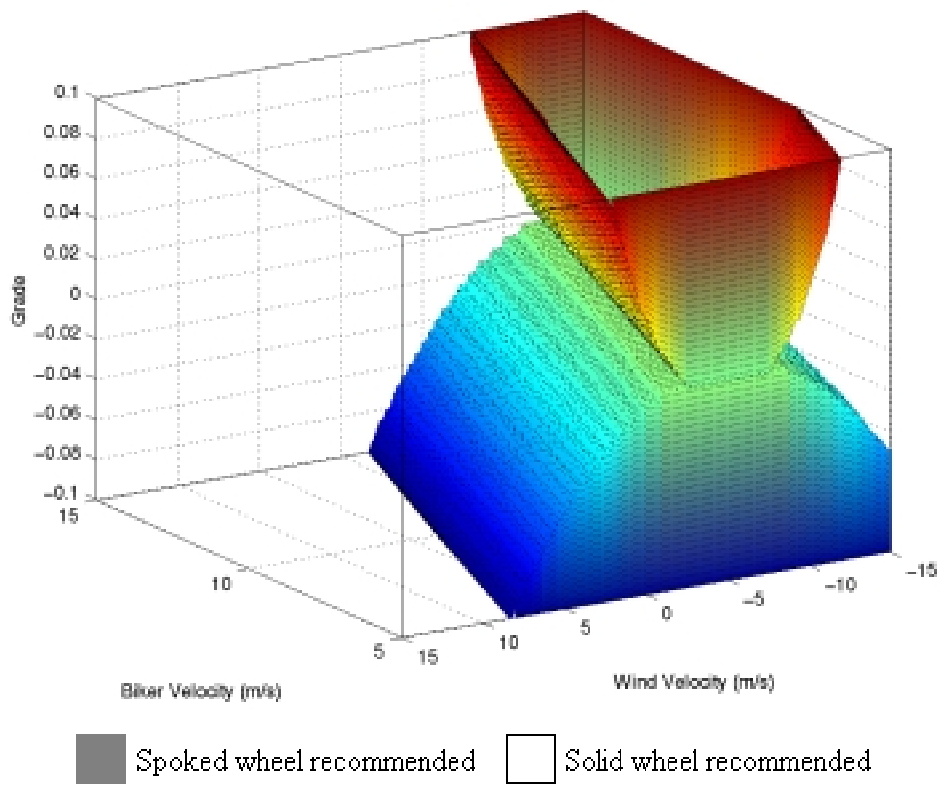

Figure 4: Environments contained in the solid correspond to a spoked wheel recommendation

3.2 Numerical Solution for Wheel Recommendation

The goal of this method is to find a regime in the (v_w, Δ) plane for which a solid rear wheel can be mathematically recommended. However, this method does not concern itself with power efficiency. Instead, this method bases its recommendation on numerous computational experiments using varied parameters. Once again, a systematic analysis of experimentally determined regimes leads to rear wheel recommendations.

To solve the equation of bicycle motion, the following two-equation system was solved numerically:

$$\begin{aligned} v' &= \frac{1}{M + 4\frac{I_{fw}}{d_{fw}^2} + 4\frac{I_{rw}}{d_{rw}^2}}[\frac{P(t)}{v} - \\ &\quad Mg\Delta - \frac{1}{2}Ac_w\rho(v+v_w)|v+v_w| - C_{rf}Mg - \\ &\quad \frac{\pi}{8}\rho(v+v_w)^2(C_{x0fw}d_{fw}^2 - \frac{3}{4}C_{x0rw}d_{rw}^2)] \end{aligned} \tag{4}$$

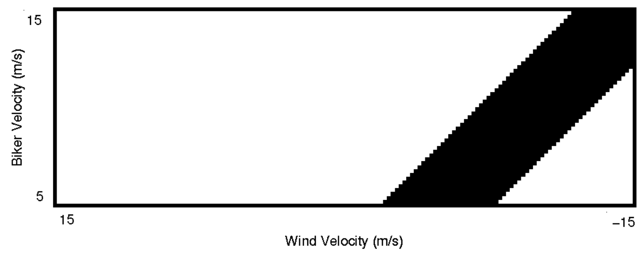

Figure 5: A horizontal slice of Figure 4 considering grade = 0.5%.

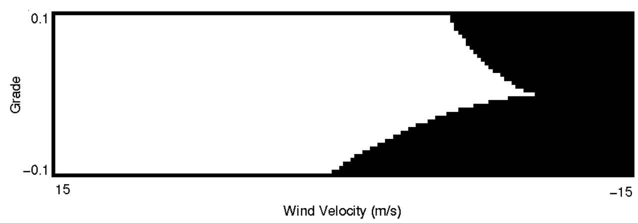

Figure 6: A vertical slice of Figure 4 considering bike velocity = 12.5 m/s.

$$x' = v \tag{5}$$

Equation (4) was solved using the fourth-order Runge-Kutta method, in intervals dt on the order of $\frac{1}{100}$ seconds. Interestingly, for a wide range of initial conditions, Euler's method (a first-order numerical scheme) performed with the same degree of accuracy as Runge-Kutta. For slowly varying velocities this may have been expected, but at times for which the cyclist first begins to accelerate from small velocities this result was surprising. To increase our confidence in the numerical scheme, we used the following diagnostic tool: we compared the results of testing two wheels with equivalent mass, rotational moment of inertia, and radius, but with variable wheel drag coefficients. Wind speeds and hill grade were varied for this diagnostic test. In this scenario, the numerical solution of equation (4) returned a velocity for the lower drag coefficient wheel that was always greater than or equal to that of the higher drag coefficient wheel. This corresponded to physical intuition, so we gained confidence in the numerical scheme.

Equation (5) was solved by numerical integration using Simpson's method. Assuming an initial position on the racetrack of 0 m, ($x(t=0)=0$),

$$x(t) = \int_0^\tau v(\tau)d\tau$$

The recommendations from this method are based on experimental trials. In consideration of computational time restraints, we focused on one course length with varying wind speeds and road grades. Doing so required some assumptions.

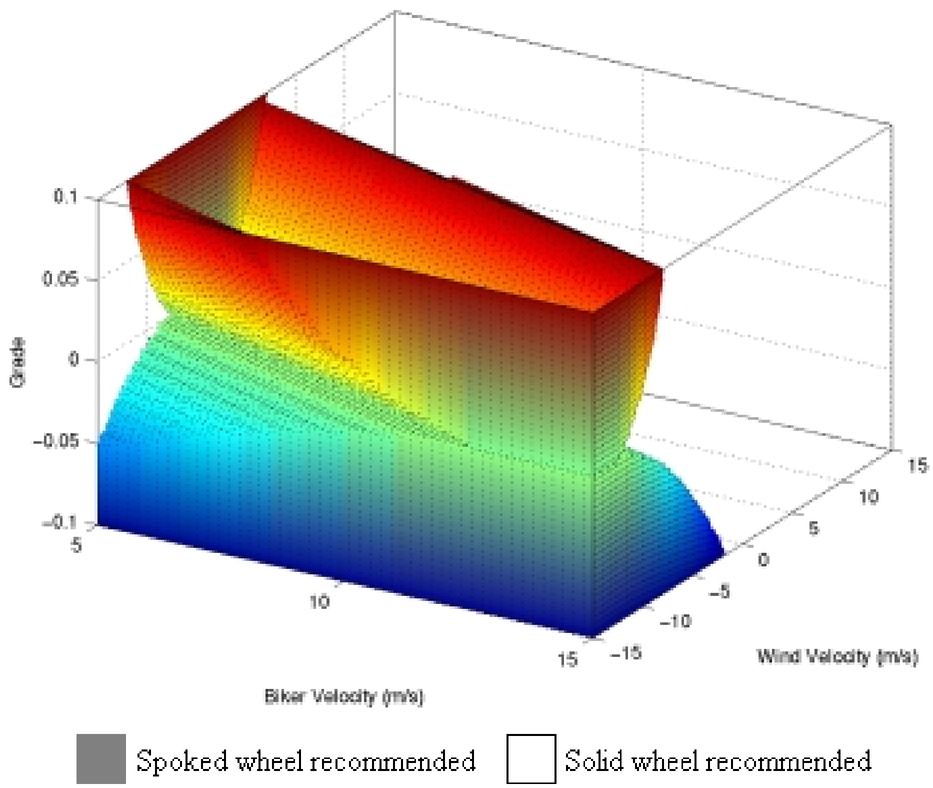

Figure 7: A Reverse View of Figure 4 - Algebraic Results

3.2.1 Method Specific Assumptions

- The experimental results for 300 m races were considered. The method assumes that the "faster" rear wheel for 300 m races will benefit a cyclist for all course lengths.
- The method assumes that the cyclist power input for this course length is constant. For experimentation, we chose a constant power $P(t) = 350$.

With these assumptions, both rear wheel configurations were tested at varying wind velocities (v_w from -15 to 15 m/s) and varying hill grades (Δ from -10% to 10%). The results of this numerical experiment are now discussed.

3.2.2 Method Results and Interpretation

The results of experimentation using this method is graphically described in figure (8) below. Once again, environmental settings in which a wire spoked wheel is recommended by this method are shaded.

	Wind Speed (m/s)	
Hill Grade	Slower 8 m/s	Sprinting 12.5 m/s
0%	(-15, -10.51)∪(-5.49, 15)	(-9.95, 15)
1%	(-15, -11.40)∪(-4.60, 15)	(-9.08, 15)
2%	(-15, -12.09)∪(-3.91, 15)	(-8.43, 15)
3%	(-15, -12.62)∪(-3.38, 15)	(-7.87, 15)
4%	(-15, -13.10)∪(-2.90, 15)	(-7.41, 15)
5%	(-15, -13.50)∪(-2.51, 15)	(-6.98, 15)
6%	(-15, -13.87)∪(-2.13, 15)	(-6.64, 15)
7%	(-15, -14.18)∪(-1.82, 15)	(-6.32, 15)
8%	(-15, -14.46)∪(-1.54, 15)	(-6.02, 15)
9%	(-15, -14.73)∪(-1.27, 15)	(-5.77, 15)
10%	(-15, -14.97)∪(-1.03, 15)	(-5.54, 15)

Table 1: Table of wind velocity regimes (in the interval), for which the algebraic method recommends a switch to a solid wheel.

This image takes into account varying cyclist speeds during the race, so there is no need for a three dimensional consideration as in the previous method results. The results are not unreasonable; at high headwinds, the aerodynamic benefit of the solid wheel will significantly affect the bicycle's performance. However, for increased tailwinds, the solid wheel's increased mass will reduce the beneficial effect of the wind pushing at the cyclist's back.

3.3 A Comparison of Recommendation Methods

It is evident from the 39 test cases run that between the two methodss, good recommendations can indeed be made: of the 39 test cases listed in Appendix A, the wheel effects in 18 of them were correctly predicted by both models (46.15 %), 11 were correctly predicted by the algebraic method when the numerical method was incorrect (28.21 %), and 10 were correctly predicted by the numerical method when the algebraic method was incorrect (25.64 %). Various negatives of each method are readily apparent. Using the numerical method, the predictions were based on the outcome of a 300 meter race at specified grade and wind velocities. Therefore, predictions made by the numerical method suffer from incorrectly adressing distance factors. This can be seen in the results of the $\Delta = 0.02$ and $v_w = -5$ m/s test case. For the 500 m race, the numerical method made a successful recommendation. However, when the distances increased to 11000 m and 50000 m, the algebraic method made the successful prediction over the numerical method.

Conversely, the algebraic method suffers from some stringent assumptions. For example the assumed equation for $\frac{dv}{dt}(\Delta, v(t))$ is based on one provided data point. Because the true equation could not be assertained, it is impossible to make statements regarding systematic error made by the algebraic method.

The remaining differences in the predictions exhibit no immediate trend. For example, both

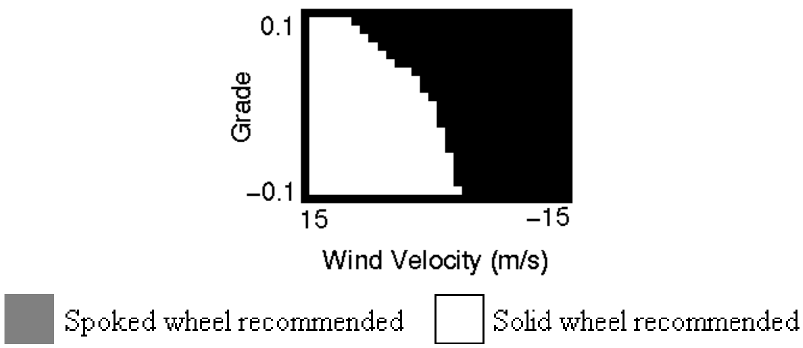

Figure 8: Numerical Method Results.

models made errors for races with varying average speeds (see the test cases with $\Delta = 0.00$, $v_w = -2$ m/s; $\Delta = 0.00$, $v_w = 10$ m/s; $\Delta = 0.10$, $v_w = -15$ m/s; and $\Delta = 0.03$, $v_w = -7$ m/s). This highlights the complexity of the dynamics at hand, where statements of systematic behavior are extraordinarily difficult to make.

One particular test run was a numerical simulation of The USCF Masters National Road Cycling Championships, a race which is to be held in July of 2001 in Spokane, Washington. Average wind velocities for Spokane in July were located [6], and the grade considered was 0.00%; the championships are held on a considerably "flat", 11,000 meter course. The results of this test are the last trials in Appendix A. For this course, the algebraic model correctly predicted which rear wheel to use: the disc wheel, in all 4 tested wind velocities. Thus, a cyclist preparing to race this course should query our algebraic model for a real wheel recommendation.

4 Model Evaluation

4.1 Strengths

The mathematical model developed in this study finds strength in its variety. By considering two methods, each with different perspectives and "opinions" about important variables, the model gains multiple views of the problem. For example, the algebraic method values power requirements, while the numerical method values speed given constant power input. The scope of the model is therefore increased. Similarly, the fact that the two recommendations produced by each method were relatively comparible increases confidence in the accuracy of each. Both are physically intuitive results, and having two sets of results from different viewpoints augments the probability that the mathematical model represents the physical situation with good precision.

Another strength of the model is the inclusion of widely varying values of wind velocity and hill grade. Larger variable ranges introduce an awareness regarding general trends.

4.2 Weaknesses / Recommendations for Further Studies

Any mathematical representation of the physical world contains assumptions, even if the assumptions are simply that of paradigm validity. Generally, we hope that our assumptions do not significantly affect our results, and that we can proceed to extract valuable information about the world from mathematical machinery. Unfortunately, not all assumptions are inconsequential, and we must consider the negative effects that our assumptions may have had on our model.

The most questionable assumption made in this study was the independence of yaw angle (ϕ) on wheel aerodynamics. Yaw angle has already been experimentally verified as significant to wheel aerodynamics [1], but due to problem complexity, we were simply left to hope for similar effects on both types of rear wheel. A recommendation for extended research of this problem is to consider variable yaw angles. New trends might appear for different wind attack angles, adding to the legitimacy of the methods developed here.

Another weakness in the mathematical model is the lack of comparison to a real world data set. There is no experimental data considered to support our conclusions, so the results of this study remain in a hypothetical state. Before reporting these findings to the cycling community, we would want to test our mathematical recommendation hypotheses in numerous physical experiments.

The algebraic method's interpolated acceleration function seems reasonable, but the mathematics are based on our own physical intuitions; the error in using such strategies is proportional to our experience of the physical world around us.

The numerical method developed failed to incorporate varying wind velocities and road grades for individual racecourses for this study. Further research would include even more realistic trials with varying wind velocities, and the physiological energy affects of longer races.

5 Conclusion

Many problems in mathematical modeling are so complex that they are often poorly understood or too convoluted to succinctly state and solve. Understanding the aerodynamic effects of different bicycle wheels in a professional cycling race is perhaps a problem that falls into this category.

Believing that the assumptions made in developing the equation were reasonable, we modelled the problem mathemtically by analyzing the motion equation in two ways. First, we algebraically manipulated the equation so that we could compare the power requirement differences between the wire spoked and solid disk rear wheels. To accomplish this task, we had to make additional assumptions about the acceleration normally experienced during a bike race. The second method of analysis was to numerically solve the force equation through time. We approached this task using two well-developed numerical methods: Euler's and Runge-Kutta. Amazingly, our Euler approximation, proven to be less accurate, performed nearly identically to the Runge-Kutta approximation. This result provides some insight into the force equation–it is very stable within the normal variable values of a bike race.

Comparing the results from our two analyses of the force equation, we found that one and generally both of the methods provided successful recommendations for which rear wheel to use. Unfortunately, we did not have the time to test the results against truly dynamic race courses–where grade changes continuously and wind velocity is not constant or parallel to the direction of bicycle motion.

References

[1] Greenwell, D. I., Bridge, E. K. L., and Addy, R. J. March 1995. "Aerodynamic characteristics of low-drag bicycle wheels." *Aeronautical Journal.* 109-120.

[2] http://www.analyticcycling.com/WheelsAeroConcept_Disc.html

[3] http://www.clubcycliste.com/english/archives/freewheel98/freew498/f498aero.html

[4] http://www.dallasnews.com/sports_day/othersp/0726othersptourup.htm

[5] http://damonrinard.com/wheel/grignon.htm

[6] http://www.ncdc.noaa.gov/ol/climate/online/ccd/avgwind.html

A Experimental Data

Several test cases were run using the numerical approximation solution to the model. The results are printed below:

Test Conditions	Course Time for Spoke Wheel	Course Time for Disc Wheel	Average $v(t)$	Predicted Outcome for Disc Wheel
$\Delta = 0.00$, $v_w = 1$ m/s Power = 350 W				Algebra: win Numerics: win
500 m	44.63 s	**44.51** s	$\approx$ 11.6 m/s	Both
11000 m	950.8 s	**947.0** s	$\approx$ 11.6 m/s	Both
50000 m	4317 s	**4299** s	$\approx$ 11.6 m/s	Both
$\Delta = -0.01$, $v_w = 1$ m/s Power = 350 W				Algebra: win Numerics: win
500 m	41.78 s	**41.66** s	$\approx$ 12.7 m/s	Both
11000 m	871.7 s	**867.6** s	$\approx$ 12.7 m/s	Both
50000 m	3954 s	**3935** s	$\approx$ 12.7 m/s	Both
$\Delta = 0.00$, $v_w = -2$ m/s Power = 350 W				Algebra: win Numerics: lose
500 m	40.18 s	**40.12** s	$\approx$ 13.5 m/s	Algebra
11000 m	815.9 s	**813.2** s	$\approx$ 13.5 m/s	Algebra
50000 m	3697 s	**3684** s	$\approx$ 13.5 m/s	Algebra
$\Delta = 0.06$, $v_w = 5$ m/s Power = 350 W				Algebra: win Numerics: lose
500 m	88.28 s	**88.22** s	$\approx$ 5.5 m/s	Algebra
11000 m	2052.9 s	**2052.7** s	$\approx$ 5.5 m/s	Algebra
50000 m	9350.1 s	**9349.5** s	$\approx$ 5.5 m/s	Algebra
$\Delta = 0.00$, $v_w = 2$ m/s Power = 350 W				Algebra: win Numerics: win
500 m	46.48 s	**46.33** s	$\approx$ 11.0 m/s	Both
11000 m	1003.4 s	**999.2** s	$\approx$ 11.0 m/s	Both
50000 m	4558 s	**4539** s	$\approx$ 11.0 m/s	Both
$\Delta = 0.05$, $v_w = -8$ m/s Power = 350 W				Algebra: lose Numerics: lose
500 m	**54.69** s	54.86 s	$\approx$ 8.75 m/s	Both
11000 m	**1249.5** s	1255.0 s	$\approx$ 8.75 m/s	Both
50000 m	**5688** s	5713 s	$\approx$ 8.75 m/s	Both

Test Conditions	Course Time for Spoke Wheel	Course Time for Disc Wheel	Average $v(t)$	Predicted Outcome for Disc Wheel
Δ = 0.08, v_w = -8 m/s Power = 350 W				Algebra: lose Numerics: lose
500 m 11000 m 50000 m	**80.26** s **1913.1** s **8721** s	80.57 s 1921.4 s 8759 s	≈ 5.8 m/s ≈ 5.8 m/s ≈ 5.8 m/s	Both Both Both
Δ = 0.08, v_w = -5 m/s Power = 350 W				Algebra: lose Numerics: lose
500 m 11000 m 50000 m	**81.53** s **1937.5** s **8831** s	81.85 s 1946.4 s 8872 s	≈ 5.7 m/s ≈ 5.7 m/s ≈ 5.7 m/s	Both Both Both
Δ = 0.00, v_w = 10 m/s Power = 350 W				Algebra: win Numerics: lose
500 m 11000 m 50000 m	70.61 s 1615.2 s 7353 s	**70.15** s **1604.8** s **7305** s	≈ 6.9 m/s ≈ 6.9 m/s ≈ 6.9 m/s	Algebra Algebra Algebra
Δ = 0.02, v_w = -5 m/s Power = 350 W				Algebra: win Numerics: lose
500 m 11000 m 50000 m	**42.63** s 873.7 s 3960 s	42.64 s **873.2** s **3957**	≈ 12.6 m/s ≈ 12.6 m/s ≈ 12.6 m/s	Numerics Algebra Algebra
Δ = 0.03, v_w = -7 m/s Power = 350 W				Algebra: win Numerics: lose
500 m 11000 m 50000 m	**44.51** s **922.4** s **4182** s	44.57 s 923.6 s 4188 s	≈ 11.9 m/s ≈ 11.9 m/s ≈ 11.9 m/s	Numerics Numerics Numerics
Δ = 0.05, v_w = -2 m/s Power = 350 W				Algebra: win Numerics: lose
500 m 11000 m 50000 m	**61.69** s **1422.1** s **6475** s	61.77 s 1424.9 s 6488 s	≈ 7.75 m/s ≈ 7.75 m/s ≈ 7.75 m/s	Numerics Numerics Numerics
Δ = 0.10, v_w = -15 m/s Power = 350 W				Algebra: win Numerics: lose
500 m 11000 m 50000 m	**85.73** s **2012.8** s **9170** s	85.89 s 2017.1 s 9190 s	≈ 5.5 m/s ≈ 5.5 m/s ≈ 5.5 m/s	Numerics Numerics Numerics

USCF Championship Race Conditions	Course Time for Spoke Wheel	Course Time for Disc Wheel	Average $v(t)$	Predicted Outcome for Disc Wheel
$\Delta = 0.00$, $v_w =$ -4 m/s Power = 350, 11000 m W				Algebra: win Numerics: lose
11000 m	742.1 s	**740.0** s	$\approx$ 15 m/s	Algebra
$\Delta = 0.00$, $v_w =$ 4 m/s Power = 350 W				Algebra: win Numerics: win
11000 m	1122.2 s	**1116.9** s	$\approx$ 11 m/s	Both
$\Delta = 0.00$, $v_w =$ -1.79 m/s Power = 350 W				Algebra: win Numerics: lose
11000 m	824.4 s	**821.6** s	$\approx$ 13.5 m/s	Algebra
$\Delta = 0.00$, $v_w =$ 1.79 m/s Power = 350 W				Algebra: win Numerics: win
11000 m	992.0 s	**987.9** s	$\approx$ 11.2 m/s	Both