Solution Derivations for Capa #9

1) Assume that a lightning bolt can be represented by a long straight line of current. If 15.0 C of charge passes by in a time of 1.5×10^{-3} s, what is the magnitude of the magnetic field at a distance of 28.0 m from the bolt?

$Q =$ Given
$t =$ Given
$r =$ Given

Current is defined to be $I = \frac{dQ}{dt}$. We know that a total charge of Q passed by in a time t, so the current is $I = \frac{Q}{t}$. The magnetic field of an infinite line is given by

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 Q}{2\pi r t}$$

2) A small diameter, 21 cm long solenoid has 263 turns and is connected in series with a resistor and battery. The size of the magnetic field inside the solenoid is 6.662×10^{-4} T when a voltage of 80.0 V is applied to the circuit. Calculate the resistance of the circuit. Use units of "Ohm".

$l =$ Given
$N =$ Given
$B =$ Given
$V =$ Given
$R =$?

The B-field of a solenoid is given by

$$B = \mu_0 n I = \frac{\mu_0 N I}{l}$$

where the last step follows since $n = \frac{N}{l}$. From Ohm's Law,

$$I = \frac{V}{R}$$

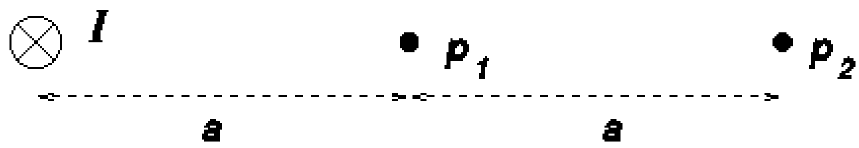
Thus,

$$B = \frac{\mu_0 N V}{l R}$$

So,

$$R = \frac{\mu_0 N V}{l B}$$

3) The wire in the above figure carries a current I which travels into the paper. North is up, East is right, etc. (For each statement select T True, F False). WARNING: you have only 4 tries.



QUESTION:
A) The magnetic field at p_1 points east.
B) The magnetic field at p_1 points south.
C) The magnetic field at p_1 points out of the paper.

ANSWER:
A) False, Use the right hand rule. Point your thumb in the direction of I, your fingers will curl in the direction of B. It is south.
B) True, See (A)
C) False, See (A)

4) If the magnetic field at point p_1 is 2.5 tesla, what is the magnetic field at p_2?

B_{p_1} = Given
B_{p_2} = ?

The magnetic field due to a long, straight wire is

$$B = \frac{\mu_0 I}{2\pi r}$$

We are told this field at point p_1.

$$B_{p_1} = \frac{\mu_0 I}{2\pi r}$$

At point p_2, the distance r is twice that at p_1. Thus,

$$B_{p_2} = \frac{\mu_0 I}{2\pi (2r)} = \frac{1}{2}\frac{\mu_0 I}{2\pi r} = \frac{1}{2} B_{p_1}$$

5) In the figure below, the bottom wire carries a current of 4.0 A. The current of the upper wire is directed in the opposite direction, and the total magnetic field at point p_1 is zero. What is the current in the top wire?

I_2 = Given
B_{p_1} = Given (0)

Since we know the field at p_1, we can find other unknowns. Equating the field due to each wire at p_1 and setting the equations equal to each other,

$$B_2 = \frac{\mu_0 I_2}{2\pi a}$$
$$B_1 = \frac{\mu_0 I_1}{2\pi (3a)}$$

Thus,

$$\begin{aligned}
\frac{\mu_0 I_2}{2\pi a} &= \frac{\mu_0 I_1}{2\pi (3a)} \\
I_2 &= \frac{I_1}{3} \\
I_1 &= 3I_2
\end{aligned}$$

6) Given that the wires are separated by $2a$, where $a = 0.021\ m$, what is the magnitude of the magnetic field at point p_2?

$a =$ Given

Using the right hand rule, the B-fields due to each wire point into the page at point p_2. Thus, we can add them together.

$$\begin{aligned}
B_{p_2} &= B_1 + B_2 \\
B_1 &= \frac{\mu_0 I_1}{2\pi a} \\
B_2 &= \frac{\mu_0 I_2}{2\pi a} \\
B_{p_2} &= \frac{\mu_0 I_1}{2\pi a} + \frac{\mu_0 I_2}{2\pi a} = \frac{\mu_0}{2\pi a}(I_1 + I_2)
\end{aligned}$$

7) In the figure below, a long straight wire carries a current of $I_a = 5.0\ A$. A square loop of edge length $0.25\ m$ is placed a distance of $0.10\ m$ away from the wire. The square loop carries a current $I_b = 2.5\ A$. Find the magnitude of the net force on the square loop.

$I_a =$ Given
$I_b =$ Given
$L =$ Given (edge length)
$d =$ Given (loop separation)

From symmetry, it can be argued that the force on the sides of the loop below will cancel each other out. Using the right hand rule, the B-Field everywhere on the loop is into the page. The total force on the loop will be that due to the top portion and that of the bottom portion. From the right hand rule, using the equation $F = IL \times B$, the bottom portion of the loop will feel an attractive force while the top portion will be repelled. Choosing up to be a positive direction,

$$F_{net} = F_{bot} - F_{top}$$

Since the wire, the B-Field, and the magnetic force are orthogonal (all perpendicular),

$$F = IL \times B = ILB$$

Thus,

$$F_{net} = I_b L B_{bot} - I_b L B_{top}$$

Since the wire is long and straight,

$$\begin{aligned}
B_{bot} &= \frac{\mu_0 I_a}{2\pi r} = \frac{\mu_0 I_a}{2\pi (d+L)} \\
B_{top} &= \frac{\mu_0 I_a}{2\pi r} = \frac{\mu_0 I_a}{2\pi d} \\
F_{net} &= I_b L \frac{\mu_0 I_a}{2\pi (d+L)} - I_b L \frac{\mu_0 I_a}{2\pi d} = I_b L \frac{\mu_0 I_a}{2\pi} \left(\frac{1}{d+L} - \frac{1}{d} \right)
\end{aligned}$$

8) A single-coil loop of radius $r = 7.10\ mm$, shown below, is formed in the middle of an infinitely long, thin, insulated straight wire carrying the current $i = 47.0\ mA$. What is the magnitude of the magnetic field at the center of the loop?

r = Given
I = Given

Since the magnetic field obeys the laws of superposition, we can add the fields due to each of the components (the line and the loop). From the book, the B-Field of an infinite line is

$$B_{line} = \frac{\mu_0 I}{2\pi r}.$$

From the lecture notes, the B-Field of a loop is

$$\begin{aligned}
B_{loop} &= \frac{\mu_0 I}{2r} \\
B_{tot} &= B_{line} + B_{loop} \\
&= \frac{\mu_0 I}{2\pi r} + \frac{\mu_0 I}{2r} = \frac{\mu_0 I}{2r} \left(\frac{1}{\pi} + 1 \right)
\end{aligned}$$

9) The four wires that lie at the corners of a square of side $a = 3.80\ cm$ are carrying equal currents $i = 2.30\ A$ into (+) or out of (-) the page, as shown in the picture. Calculate the y component of the magnetic field at the center of the square.

a = Given
I = Given

Using the right hand rule, we can see that the magnetic field from the top left and bottom right wires points in the upper right (the (+) direction is into the page). The other two corners have their magnetic field pointing in the upper left. The x-components for each B-Field vector will cancel leaving only the y-component determining the magnitude. From symmetry, the magnetic field from each wire is equal at the center. Thus, the total B-Field is four times that due to one of the wires. Also from symmetry, the angle each vector makes with the y-axis is 45°.

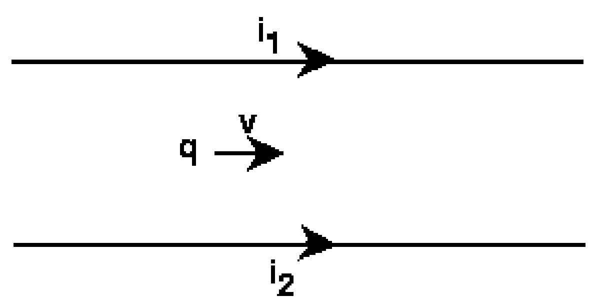
Thus, it does not matter whether we use the sine or cosine to find the vertical component.

$$B = 4\frac{\mu_0 I}{2\pi r}\sin 45^\circ = 4\frac{\mu_0 I}{2\pi r}\frac{1}{\sqrt{2}} = 4\frac{\mu_0 I}{2\sqrt{2}\pi r}$$

Again from symmetry, r is the length of the hypotenuse whose side lengths are both $\frac{a}{2}$. Thus,

$$\begin{aligned} r &= \sqrt{2}\frac{a}{2} \\ B &= 4\frac{\mu_0 I}{2\sqrt{2}\pi\sqrt{2}\frac{a}{2}} = 4\frac{2\mu_0 I}{4\pi a} = \frac{2\mu_0 I}{\pi a} \end{aligned}$$

10) In the picture below, the two wires carry current i_1 and i_2 respectively, with positive current to the right. The charge q, midway between the wires, is positive and has velocity v to the right. North is up, East is to the right, etc. Which of the following statements for magnetism are true? (Give ALL correct answers in alphabetical order, i.e., B, AC, BCD...)



QUESTION:
A) If $i_1 > i_2$, then the force on q is north.
B) If $i_2 = 0$ and $i_1 > 0$, then the magnetic field near charge q points out of the page.
C) If $v = 0$, then the force on q is zero.
D) If $i_1 = -i_2$, then the force on q is zero.
E) If $i_1 = 0$ and $i_2 > 0$, then the force on q points out of the page.
F) If $i_1 = 0$ and $i_2 > 0$, then the force on q is south.
G) If $i_1 = i_2$, then the wires are attracted to each other.

ANSWER:
A) True, The top wire produces a magnetic field into the page at q. The bottom wire produces a magnetic field pointing out of the page at q. If the top current is stronger, the force will be north (from $qv \times B$).
B) False, The top wire produces a magnetic field into the page at q (right hand

rule).
C) True, Magnetic forces are created by moving charges ($qv \times B$).
D) False, In this case, both wires produce a magnetic field out of the page at q and the force is downward.
E) False, The bottom wire produces a magnetic field pointing out of the page at q. Thus, the force is south ($qv \times B$).
F) True, See (E).
G) True, parallel currents attract (see lecture notes 30-2).

11) A solid rod of radius R carries a uniformly distributed current $i_0 = 3.00\ A$ into the page. A wire lies a distance 2R away from the surface of the rod. What is the magnitude and direction (into (+) or out of (-) the page) of the current in the wire so that the magnetic field, $\vec{\mathbf{B}}$, at a point P, half-way between the two, equals the field at the center of the rod? (The two fields must be equal in magnitude and direction.)

$I =$ Given

The magnetic field at the center of the rod is only due to the wire (since there is no current of the rod contained in any amperian loop, there is no B-Field due to it). Choose the direction of I to be positive (into the page) to avoid any imbedded negative signs.

$$B_{center} = -\frac{\mu_0 I_{wire}}{2\pi\,(3R)}$$

This is negative since the right hand rule gives the direction as down. At point P, the B-Field is influenced by both the wire and the rod.

$$B_{P,wire} = -\frac{\mu_0 I_{wire}}{2\pi R}$$

Where again it is negative since the direction is down.

$$B_{P,rod} = \frac{\mu_0 I_{rod}}{2\pi\,(2R)}$$

Where the radius is $2R$ since it is measured from the center of the rod (just like the radius from a solid sphere of charge was taken from the center). It is positive because the right hand rule gives the direction as up. Equating the B-Field at point P to that at the center of the rod allows us to solve for the unknown current.

$$\begin{aligned}
B_{center} &= B_{P,wire} + B_{P,rod} \\
-\frac{\mu_0 I_{wire}}{2\pi\,(3R)} &= -\frac{\mu_0 I_{wire}}{2\pi R} + \frac{\mu_0 I_{rod}}{2\pi\,(2R)} \\
-\frac{\mu_0}{2\pi R}\frac{I_{wire}}{3} &= \frac{\mu_0}{2\pi R}\left(-I_{wire} + \frac{I_{rod}}{2}\right) \\
-\frac{I_{wire}}{3} &= -I_{wire} + \frac{I_{rod}}{2}
\end{aligned}$$

$$\begin{aligned}\frac{2I_{wire}}{3} &= \frac{I_{rod}}{2} \\ I_{wire} &= \frac{3}{4}I_{rod}\end{aligned}$$

12) Two very long solenoids have the same length, but solenoid A has 17 times the number of turns, 1/9 the radius, and 4 times the current of solenoid B. Calculate the ratio of the magnetic field inside A to that inside B.

Given:
$L_A = L_B$
$N_A = xN_B$
$r_A = yr_B$
$I_A = zI_B$
where x, y, z are proportionality constants.

The B-Field of a solenoid is given by

$$B = \mu_0 nI$$

where $n = N/L$. Note that the radius is nowhere in the equation. The ratio wanted is that of the field in A to that in B. Thus,

$$\frac{B_A}{B_B} = \frac{\mu_0 \frac{N_A}{L} I_A}{\mu_0 \frac{N_B}{L} I_B} = \frac{\mu_0 \frac{xN_B}{L} zI_B}{\mu_0 \frac{N_B}{L} I_B} = x * z$$

13) The figure shows a hollow cylindrical conductor with radii $a = 1.1\ cm$ and $b = 4.2\ cm$ which carries a current $5.0\ A$ uniformly spread over its cross-section. Find the magnitude of the magnetic field at a distance of $2.6\ cm$ from the axis.

$a =$ Given
$b =$ Given
$I =$ Given
$r =$ Given (distance at which to calculate the magnetic field)

This is similar to concept test 30-3. From Ampère's Law,

$$\oint B \cdot dl = \mu_0 I_{enc}$$

Since the B-field is constant at a given radius and perpendicular to the length vector, it can be taken outside the integral.

$$\begin{aligned}B \oint dl &= \mu_0 I_{enc} \\ B * 2\pi r &= \mu_0 I_{enc} \\ B &= \frac{\mu_0 I_{enc}}{2\pi r}\end{aligned}$$

So the hollow rod behaves like a wire. Now we need to know the current enclosed. We can find the current density of the rod and then multiply by the area to radius r to find the current enclosed. This is only possible since the current is uniformly distributed.

$$J = \frac{I}{A} = \frac{I}{\pi b^2 - \pi a^2}$$

Where the area is just the total area of the conductor minus the area of the hollow part. The area of the conductor enclosed to the radius r is

$$A_{enc} = \pi r^2 - \pi a^2$$

Thus,

$$\begin{aligned} I_{enc} &= J * A_{enc} \\ &= \frac{I}{\pi b^2 - \pi a^2}\left(\pi r^2 - \pi a^2\right) \\ &= \frac{I\pi\left(r^2 - a^2\right)}{\pi\left(b^2 - a^2\right)} = \frac{I\left(r^2 - a^2\right)}{\left(b^2 - a^2\right)} \end{aligned}$$

So,

$$B = \frac{\mu_0 I_{enc}}{2\pi r} = \frac{\mu_0 I\left(r^2 - a^2\right)}{2\pi r\left(b^2 - a^2\right)}$$

14) A long hairpin is formed by bending an infinitely long wire, as shown. If a current of 4.8 A is set up in the wire, what is the magnitude of the magnetic field at the point a? Assume $R = 6.0\ cm$.

$I =$ Given
$R =$ Given

This problem can be thought of as consisting of half a loop of wire and two half-infinite lengths of wire (is there such a thing?). Thus,

$$\begin{aligned} B_a &= \frac{1}{2}B_{loop} + 2\left(\frac{1}{2}B_{wire}\right) \\ &= \frac{1}{2}\frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi R} \\ &= \frac{\mu_0 I}{2R}\left(\frac{1}{2} + \frac{1}{\pi}\right) \end{aligned}$$

15) Decide from the following list of possibilities what is the appropriate direction of the force on the wire for each of the diagrams 1 through 4: WARNING: you have only 8 tries for this problem.

Using the right hand rule and the equation $IL \times B$,
A) out of the page
B) zero force. The vectors are antiparallel, $IL \times B = ILB\sin\theta$, $\sin 180° = 0$
C) into the page
D) out of the page