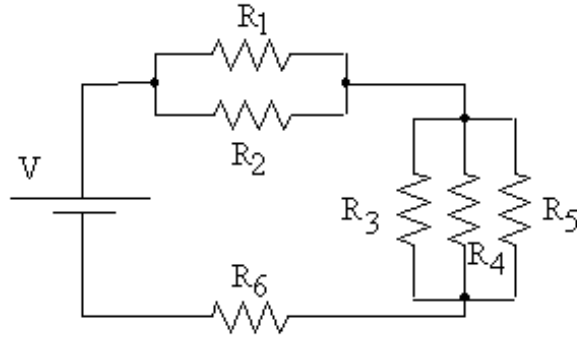


## Solution Derivations for Capa #7

1) Consider the behavior of the circuit, when various values increase or decrease. (Select I-increases, D-decreases, If the first is I and the rest D, enter IDDDD).



QUESTION:

- A) If R1 decreases, the current through R3 .....
- B) If R3 decreases, the current from the battery.....
- C) If R6 decreases, the power output from the battery .....
- D) If R1 decreases, the current through R6 .....
- E) If the voltage V decreases, the total current from the battery .....
- F) If R3 decreases, the current through R4 .....

ANSWER:

- A) Increases, The equivalent resistance of the parallel resistor decreases. From  $V = IR$ , the current will increase.
- B) Increases. Same as (A).
- C) Increases.  $P = IV$ . If the current rises due to a decrease in resistance, the power will also rise.
- D) Increases. See (A).
- E) Decreases. Less voltage to push current through the circuit.
- F) Decreases. More will want to flow through R3 with a now lower resistance.

2) A battery with an emf of 12.00 V has an internal resistance  $r$ . When connected to a resistor R the terminal voltage is 11.70 V and the current is 0.11 A. What is the value of the external resistor R? Use “Ohm” as units of resistance.

- $V_E =$  Given (EMF)
- $V =$  Given
- $I =$  Given
- $r = ?$
- $R = ?$

To solve this problem, you don't have to worry about the internal resistance. Thus, Ohm's law applies directly to the given information.

$$V = IR$$

$$R = \frac{V}{I}$$

Note that the voltage is not equal to the supplied EMF when the resistor is connected.

3) What is the internal resistance  $r$  of the battery?

Internal resistance of a battery is similar to having two resistors in series. Thus, the equivalent resistance of the circuit is simply

$$R_{tot} = r + R$$

From  $V = IR$ ,

$$R_{tot} = r + R = \frac{V_E}{I}$$

Thus, the internal resistance is

$$r = \frac{V_E}{I} - R$$

This time the total voltage supplied by the EMF is used since we are taking into account that there is internal resistance. In the previous problem, we used the remaining voltage available after the voltage drop across the internal resistor.

4) The picture shows a battery connected to two cylindrical resistors in parallel. Both resistors are made of the same material and are of the same length, but the diameter of resistor A is twice the diameter of resistor B. (For each statement select T True, F False).

QUESTION:

- A) The resistance of wire B is four times as large as the resistance of wire A.
- B) The current through the battery is five times larger than the current through wire B.
- C) The power dissipated in wire A is 16 times the power dissipated in wire B.
- D) The resistance of wire B is twice as large as the resistance of wire A.
- E) The voltage drop across wire B is larger than the voltage drop across wire A.

ANSWER:

A) True. Resistance is given by

$$R = \frac{\rho L}{A}.$$

Thus, if the area increases, the resistance decreases by the square of that factor (in the cylinder case).

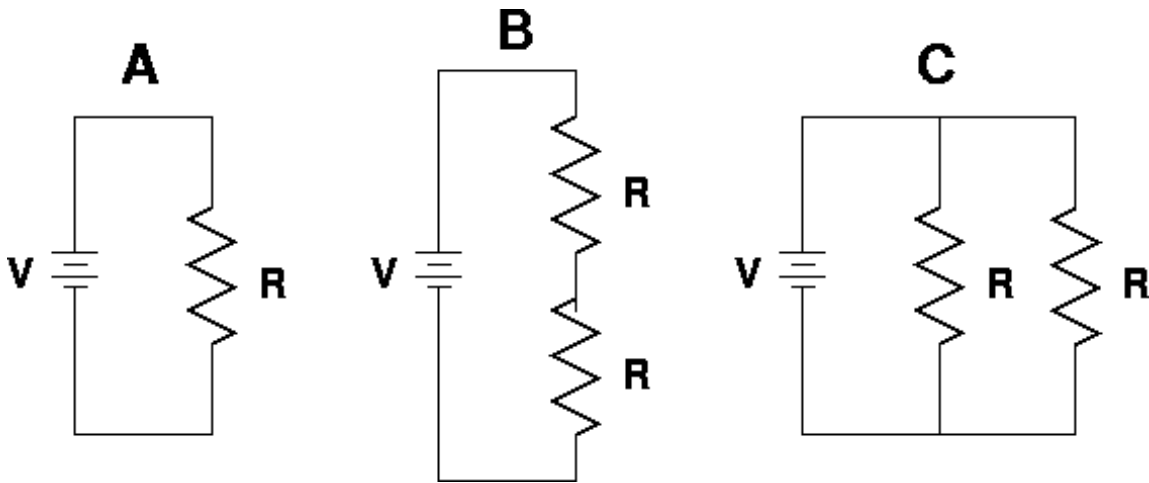
B) True. The resistance through B is four times that in A. Together, they dissipate all the current. The total current is then 5 times the current in A. (The current in A plus 4 times that current in B).

C) False,  $P = IV$ . From (E), they both have the same voltage drop. From (B), A has 4 times the current. Thus, it only dissipates 4 times the power.

D) False. See (A)

E) False. The voltage drop across a parallel configuration of resistors is equal. It must be from the definition. The wires are connected together at each end, so they must be at the same voltage. Thus, the current must split accordingly.

5) Consider the three circuits shown above. All the resistors and all the batteries are identical. (For each statement select T True, F False).



QUESTION:

- A) The power dissipated in circuit A is twice the power dissipated in circuit B.
- B) The total power dissipated in circuit C is twice the total power dissipated in circuit B.
- C) The current through a resistor is the same in circuits A and B.
- D) The current through a resistor is the same in circuits A and C.
- E) The voltage across a single resistor in circuit C is twice the voltage across a single resistor in circuit B.

ANSWER:

A) True.

$$P = \frac{V^2}{R}$$

The resistance in B is twice that in A, so A dissipates twice the power.

B) False. It is actually 4 times. The equivalent resistance of C is

$$\frac{1}{\frac{1}{R} + \frac{1}{R}} = \frac{R}{2}$$

Plugging into

$$P = \frac{V^2}{R}$$

we get

$$P = 2\frac{V^2}{R}$$

which is four times that of B:

$$P = \frac{V^2}{2R}$$

C) False. From  $V = IR$ , we get

$$I = \frac{V}{R}$$

Since the resistance is greater in B, it has less current.

D) True. The current in C is twice that in A, since

$$I = \frac{V}{R}$$

and the resistance in C is

$$\frac{R}{2}$$

But, it is split among two equal resistors. So, each carries half that amount of current. Thus, the current through a resistor in C is the same as the current through the resistor in A.

E) True. Each resistor in C has the entire voltage drop across it. However, circuit B has two equal resistors in series, so the voltage must drop across each of them. Since they are equal resistance, the voltage drops half of its full amount across each one.

Remember to answer in the form TFFTT

6) Consider the sections of two circuits illustrated above. All resistances shown have a finite, non-zero, resistance. (Give ALL correct answers, i.e., B, AC, BCD...) WARNING: You have only 8 tries for this problem.

QUESTION:

- A)  $R_{cd}$  is always less than  $R_3$ .
- B)  $R_{ab}$  is always less than  $R_1$ .
- C) After connecting  $c$  and  $d$  to a battery, the current through  $R_3$  always equals the current through  $R_4$ .
- D) After connecting  $a$  and  $b$  to a battery, the voltage across  $R_1$  always equals the voltage across  $R_2$ .

ANSWER:

- A) False, Resistors in series always add to a greater resistance than either one

individually.

B) True, Resistors in parallel always add to a lesser resistance than either one individually.

C) True, The current is the same for all resistors in series as it has no where else to go.

D) True, The voltage is the same for all resistors in parallel because each of their ends are connected to each other and the voltage must be equal on both sides.

Remember to answer this one in the form BCD.

7) In the section of circuit below,  $R_1 = 16.489 \Omega$ ,  $R_2 = 1.120 \Omega$  and the voltage difference  $V_a - V_b = 2.400 V$ . The current  $i = 0.140 \text{ amps}$ . Find the value of  $R_3$ . Use “Ohm” as your units.

$R_1 = \text{Given}$

$R_2 = \text{Given}$

$V = \text{Given}$

$I = \text{Given}$

We must find the equivalent resistance of the diagram. There is one set in parallel and one other resistor in series. Thus, we can add the two together to get

$$R_{tot} = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

From  $V = IR$ ,  $R = \frac{V}{I}$

$$\begin{aligned} R &= R_1 + \frac{R_2 R_3}{R_2 + R_3} = \frac{V}{I} \\ \frac{R_2 R_3}{R_2 + R_3} &= \frac{V}{I} - R_1 \\ R_2 R_3 &= \left( \frac{V}{I} - R_1 \right) (R_2 + R_3) \\ &= \frac{V}{I} R_2 + \frac{V}{I} R_3 - R_1 R_2 - R_1 R_3 \\ R_2 R_3 + R_1 R_3 - \frac{V}{I} R_3 &= \frac{V}{I} R_2 - R_1 R_2 \\ R_3 \left( R_2 + R_1 - \frac{V}{I} \right) &= \frac{V}{I} R_2 - R_1 R_2 \\ R_3 &= \frac{\frac{V}{I} R_2 - R_1 R_2}{R_2 + R_1 - \frac{V}{I}} \end{aligned}$$

8) What is the current through  $R_3$ ?

We are given the current and it must all travel through the diagram. If we find the voltage drop across the first resistor, we know the remaining voltage must drop across the second resistor and can then find the current.

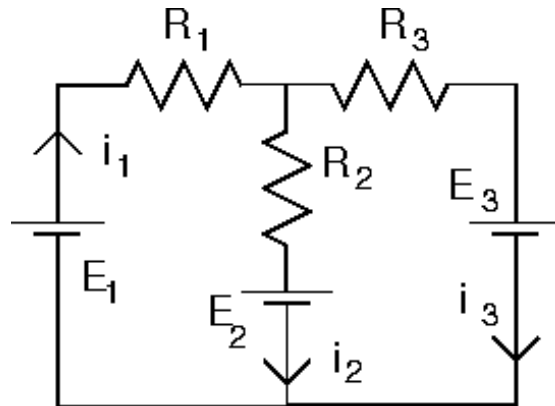
$$V_1 = IR_1$$

Since this is the voltage drop across the first resistor, there is still  $V - V_1$  total voltage to drop over the parallel resistors. Thus,

$$V_3 = V - V_1 = IR_3$$

$$I = \frac{V - V_1}{R_3}$$

9) Consider the following diagram. (Give ALL correct answers in alphabetical order, i.e., B, AC, BCD...)



QUESTION:

- A)  $E_1 - E_2 = i_1 R_1 + i_2 R_2$
- B)  $i_1 + i_2 = i_3$
- C)  $i_1 = i_2 + i_3$
- D)  $E_2 - E_3 = i_3 R_3 - i_2 R_2$
- E)  $E_1 - E_3 = i_1 R_1 - i_3 R_3$
- F)  $E_2 - E_1 = i_1 R_1 - i_2 R_2$
- G)  $E_3 - E_1 = -i_3 R_3 - i_1 R_1$

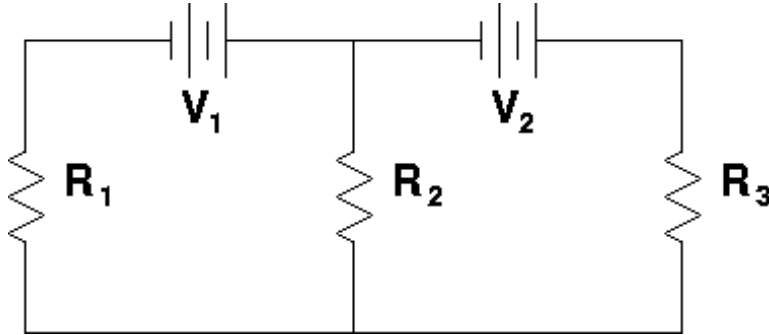
ANSWER:

- A) True
- B) False
- C) True
- D) True
- E) False
- F) False

G) True

Use Kirchoff's rules and follow a loop around the circuit. Only pay attention to the directions that are labeled, not which way you feel the current should flow. Remember to enter this question in the form ACDG.

10) What is the magnitude of the current through  $R_1$ ? DATA:  $R_1 = R_2 = R_3 = 16.0 \Omega$ .  $V_1 = 12.0$  Volts.  $V_2 = 2V_1$ .



Assigning arbitrary directions to the current flow and obtaining 3 equations,

$$V_1 = -I_2 R_2 + I_1 R_1 \quad (1)$$

$$V_2 = I_3 R_3 + I_2 R_2 \quad (2)$$

$$I_3 = I_1 + I_2 \quad (3)$$

Now solving for the three unknowns with the three equations, plugging (3) into (2)

$$V_2 = (I_1 + I_2) R_3 + I_2 R_2 = I_1 R_3 + I_2 R_3 + I_2 R_2 \quad (4)$$

Solving (1) for  $I_2$  and plugging into (4),

$$V_1 = -I_2 R_2 + I_1 R_1$$

$$I_2 = \frac{I_1 R_1 - V_1}{R_2}$$

$$V_2 = I_1 R_3 + I_2 R_3 + I_2 R_2$$

$$V_2 = I_1 R_3 + \frac{I_1 R_1 - V_1}{R_2} R_3 + \frac{I_1 R_1 - V_1}{R_2} R_2$$

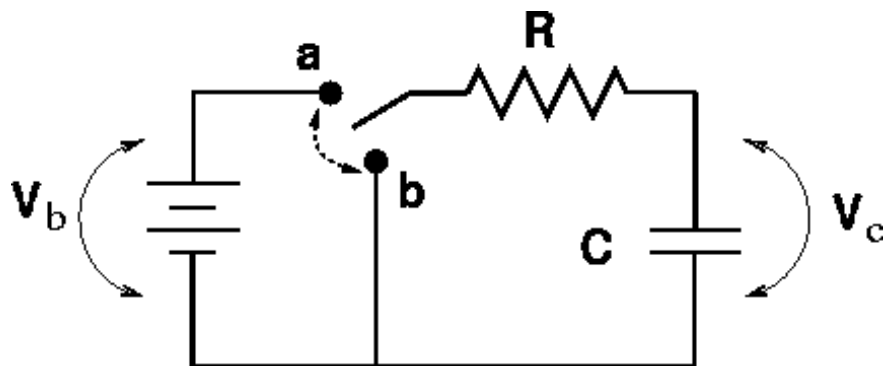
$$V_2 = I_1 R_3 + \frac{I_1 R_1 - V_1}{R_2} R_3 + I_1 R_1 - V_1$$

$$V_2 + V_1 = I_1 R_3 + (I_1 R_1 - V_1) \frac{R_3}{R_2} + I_1 R_1 = I_1 R_3 + \frac{I_1 R_1 R_3}{R_2} - V_1 \frac{R_3}{R_2} + I_1 R_1$$

$$V_2 + V_1 + V_1 \frac{R_3}{R_2} = I_1 R_3 + \frac{I_1 R_1 R_3}{R_2} + I_1 R_1 = I_1 \left( R_3 + \frac{R_1 R_3}{R_2} + R_1 \right)$$

Thus,

$$I_1 = \frac{V_2 + V_1 + V_1 \frac{R_3}{R_2}}{R_3 + \frac{R_1 R_3}{R_2} + R_1}$$



11) The circuit which was at position  $a$  for a long time is suddenly switched to position  $b$  at time  $t = 0$ . (For each statement select T True, F False). WARNING: you have 8 tries only for this question.

QUESTION:

- A) In the instant after the switch is thrown, the voltage across the capacitor is zero.
- B) In the instant after the switch is thrown the voltage across the resistor is zero.
- C) The current through the resistor equals the current through the capacitor at all times.
- D) In the instant after the switch is thrown the current through the capacitor is zero.

ANSWER:

- A) False, the capacitor is currently charged, so there is a voltage change.
- B) False, the capacitor has a voltage change across its plates so there will be a voltage across the resistor.
- C) True, the resistor and capacitor are in series; the current must always be equal.
- D) False. Charge from the capacitor is moving around the circuit, so there is a current through it.

12) What is  $V_c$  at time  $t = 2.0 \text{ ms}$ ? DATA:  $R = 475 \Omega$ ,  $C = 4.0 \mu\text{F}$ ,  $V_b = 2.95 \text{ V}$ .

- $R = \text{Given}$
- $C = \text{Given}$
- $V_b = \text{Given}$
- $t = \text{Given}$

The voltage for a discharging capacitor is given by equation 28-8 on page 710 of the textbook.

$$V = V_0 e^{-t/RC}$$

Which equates to our variables as

$$V_C = V_b e^{-t/RC}$$



13) What is the energy dissipated in the resistor after time  $t = 0$  (that is, for all time after  $t = 0$ , meaning the during the time period from  $t = 0$  to infinity.)

One of the energy equations for a capacitor is

$$U = \frac{1}{2}CV^2$$

As time goes to infinity, the entire capacitor will discharge. It was charged to the initial voltage of the battery.

14) A solid truncated cone is made of a material of resistivity  $3.50 \Omega \cdot m$ . The cone has height  $h = 1.06 m$ , and radii  $a = 0.33 m$  and  $b = 0.89 m$ . What is the total resistance for this cone? (Use “Ohm” as your units.)

$\rho =$  Given

$a =$  Given

$b =$  Given

$h =$  Given

To solve this problem we need to treat the frustum like a collection of infinitely many resistors that have the same radius over their length. In other words, we need to integrate. The equation for resistance is

$$R = \frac{\rho L}{A}$$

where  $L$  is the length of the resistor and  $A$  is the cross-sectional area. We will be taking small portions of this resistor so that the radius is the same over that length. Thus, the length will be the differential  $dx$ . We will be integrating over the length of the resistor taking the left side to be  $x = 0$  and the right side to be  $x = h$ . So far,

$$R = \int_0^h \frac{\rho}{A} dx$$

Now, we must relate  $A$  to  $dx$ . When  $x = 0$ ,  $r = a$  and when  $x = h$ ,  $r = b$ . Thus, we can think of the outer surface of the cone as a line that must go through the points  $(0, a)$  and  $(h, b)$ . Using point slope form, the slope is  $\frac{b-a}{h}$ . Thus, the equation of this line using point-slope form is

$$\begin{aligned} y - a &= \frac{b - a}{h}x \\ y &= \frac{b - a}{h}x + a \end{aligned}$$

Basically,  $y$  is simply a relation of the radius of the resistor to the length of it. Thus, it is  $r$ . The area of each of these little slices is

$$A = \pi r^2 = \pi \left( \frac{b - a}{h}x + a \right)^2$$

Substituting this back into the integral,

$$R = \int_0^h \frac{\rho}{\pi \left(\frac{b-a}{h}x + a\right)^2} dx = \frac{\rho}{\pi} \int_0^h \frac{dx}{\left(\frac{b-a}{h}x + a\right)^2}$$

Using a  $u$ -substitution of  $u = \frac{b-a}{h}x + a$ ,  $du = \frac{b-a}{h} dx$ ,  $dx = \frac{h}{b-a} du$ , we get the transformed integral

$$\begin{aligned} R &= \frac{\rho h}{\pi (b-a)} \int_a^b \frac{du}{u^2} = \frac{\rho h}{\pi (b-a)} \left( -\frac{1}{u} \Big|_{u=a}^b \right) \\ R &= -\frac{\rho h}{\pi b (b-a)} + \frac{\rho h}{\pi a (b-a)} \end{aligned}$$