Solution Derivations for Capa #14

1) A hollow sphere of inner radius $r_i = 6.00\ cm$ and outer radius $r_o = 8.60\ cm$ floats half submerged in a liquid of density $\rho_{liq} = 850\ kg/m^3$. What is the mass m of the sphere?

r_i = Given in cm.
r_o = Given in cm.
ρ_{liq} = Given

Per Archemedes, the force on an object is equal to the weight of displaced fluid. If the sphere is half-submerged, we can find the displaced volume of fluid and then its weight. From the weight we can extract the mass.

$$v = \frac{4}{3}\pi r_o^3$$

Since the sphere is half-submerged,

$$\frac{1}{2}v = \frac{2}{3}\pi r_o^3$$

is what we want. Since density is mass per unit volume,

$$\rho_{liq} = \frac{m}{v}$$

or

$$v\rho_{liq} = m$$

where v in our case is half the volume of the sphere, or

$$\frac{2}{3}\pi r_o^3.$$

$$\frac{2}{3}\pi r_o^3 \rho_{liq} = m.$$

Make sure to convert r_o to meters.

2) Calculate the density ρ of the material of which the sphere is made.

r_i = Given in cm.
r_o = Given in cm.

Again, density is mass per unit volume. The mass of this sphere was calculated in problem (1), so we can use it for this one. We just need to find the volume of the sphere. Actually, we need to find the volume of the *mass* in the sphere which we will use to find the density. The volume of this hollow sphere is the volume of the large, outer sphere minus the volume of the smaller, inner sphere. Or,

$$\begin{aligned} v_{tot} &= v_{out} - v_{in} \\ v_{tot} &= \frac{4}{3}\pi r_o^3 - \frac{4}{3}\pi r_i^3 = \frac{4}{3}\pi\left(r_o^3 - r_i^3\right) \end{aligned}$$

Using the mass from problem (1),

$$\rho = \frac{m}{v} = \frac{m}{\frac{4}{3}\pi\left(r_o^3 - r_i^3\right)} = \frac{3m}{4\pi\left(r_o^3 - r_i^3\right)}$$

Remember to convert both r_i and r_o to meters. Density has units of kg/m^3.

3) Find the initial downward acceleration a of an iron ball of density $\rho = 8.40\ g/cm^3$ when placed in water. (Use $\rho_{water} = 1000\ kg/m^3$).

$\rho =$ Given in g/cm^3
$\rho_{water} =$ Given

Assume you have one cm^3 of iron of density ρ. So you have whatever magnitude of $\frac{\rho}{1000}$ kilograms of iron (the mass, since ρ was given in g/cm^3) and 1 cm^3 of volume. The buoyant force exerted on this object is equal to the weight of the displaced water. The force is simple the weight of 1 cm^3 of water.

$$F = \rho_{water} * 1\ cm^3 * \left(\frac{1\ m}{100\ cm}\right)^3 * g = \frac{g\rho_{water}}{1000000}$$

(which has units of newtons). That is the upward force due to buoyancy. The downward force is due to the weight of the ball. The weight is simply $W = mg$ where the mass is defined above. The forces are acting in opposite directions, so pick a coordinate system and find the difference. Here, we assume that up is the positive direction.

$$F_{net} = ma = F - W$$

To find the resulting initial acceleration, simply divide the total force by the mass of the object.

$$\begin{aligned} ma &= \frac{g\rho_{water}}{1000000} - mg \\ a &= \frac{g\rho_{water}}{1000000 * m} - g \end{aligned}$$

To help solve this, just think of m as equaling $\frac{\rho}{1000}$. This quantity will have units of kg/cm^3 but then you are assuming you have 1 cm^3 of the substance, so the mass in kilograms is all that's left. This method of solving the problem will probably produce a negative acceleration, meaning downward. The problem asks for the initial downward acceleration, so take the absolute value of the answer.

4) One of the problems with debris in the ocean is that it is often difficult to see, because much of the object is under the surface of the water. An object with a density of $\rho = 751\ kg/m^3$ and a mass $m = 2443\ kg$ is thrown into the ocean. Find the fraction f of the object that is sticking out of the water (use $\rho_{seawater} = 1024\ kg/m^3$).

$\rho =$ Given
$m =$ Given
$\rho_{seawater} =$ Given

To find the volume of the object thrown in the ocean,

$$\begin{aligned} \rho &= \frac{m}{v}, \\ v &= \frac{m}{\rho} \end{aligned}$$

This problem is basically asking for what percentage of the total volume of the object is sticking out of the ocean. This can be found when the object is floating; that is, when the buoyant force equals the weight of the object. The buoyant force is the weight of displaced water which equals

$$F = \rho_{seawater} v_{displaced} g$$

The weight of the object is simply $W = mg$. Setting these equal and solving for $v_{displaced}$,

$$\begin{aligned} \rho_{seawater} v_{displaced} g &= mg \\ v_{displaced} &= \frac{m}{\rho_{seawater}} \end{aligned}$$

Computing the ratio $\frac{v_{displaced}}{v}$ will give you the percentage of the object *below* the surface. Take this number and subtract it from 1 to find the percentage above the surface.

5) A piston with a tight-fitting, but movable lid contains an ideal gas at a pressure p, an initial temperature T_i and an initial volume V_i. The temperature of the piston is slowly increased by heating the outside while the pressure is maintained constant. The temperature is raised to a final temperature of $T_f = 3.5\,T_i$. As the temperature increases, the volume changes until the final volume is $V_f = f\,V_i$, where f is a dimensionless factor. What is the factor f by which the volume changes?

$p =$ Given
$T_i =$ Given
$V_i =$ Given
$T_f =$ Given in terms of T_i
$V_f = f\,V_i$

The problem asks for the scalar f by which the volume changes. To solve this, you must use the ideal gas law.

$$PV = nRT$$

to relate the initial and final conditions.

$$P_iV_i = n_iRT_i$$

and

$$P_fV_f = n_fRT_f$$

Since the pressure does not change, we can relate the two equations by solving for the pressure and then substitute it back into the first equation. The container is sealed, so the number of gas particles (n) also does not change.

$$\begin{aligned} P &= \frac{nRT_f}{V_f} \\ \frac{nRT_f}{V_f}V_i &= nRT_i \end{aligned}$$

Simplifying,

$$T_fV_i = T_iV_f$$

Or

$$\frac{T_f}{T_i}V_i = V_f$$

Which is similar to

$$V_f = f * V_i$$

So,

$$f = \frac{T_f}{T_i}$$

6) A container of fixed volume V has a tiny hole through which gas can enter or leave. The container is filled with an ideal gas at the same temperature and pressure p as the surrounding room air. The initial temperature is T_i and the initial number of molecules of ideal gas in the container is N_i. The temperature of the container is increased by heating the outside until the final temperature is $T_f = 2.50\,T_i$. As the temperature increases, the number of molecules remaining in the container decreases until the final number is $N_f = f\,N_i$, where f is a dimensionless factor. What is the factor f by which the number of molecules changes?

V = constant
T_i = Given
n_i = Given
T_f = Given in terms of T_i
$n_f = f\,n_i$

The problem asks for the scalar f by which the number of gas particles changes. To solve this, you must use the ideal gas law.

$$P_iV_i = n_iRT_i$$

and

$$P_f V_f = n_f R T_f$$

Dividing the two equations,

$$\frac{P_i V_i}{P_f V_f} = \frac{n_i R T_i}{n_f R T_f}$$

Since the volume does not change, the pressure is allowed to equalize with the air outside the container, and R is a constant,

$$\begin{aligned} 1 &= \frac{n_i T_i}{n_f T_f} \\ n_f T_f &= n_i T_i \\ n_f &= \frac{n_i T_i}{T_f} \end{aligned}$$

Which is similar to

$$n_f = f * n_i,$$

so

$$f = \frac{T_i}{T_f}$$

7) A piston with a tight-fitting, but movable lid contains an ideal gas at an initial temperature $T_i = 280\ K$, an initial pressure $p_i = 1.00\ atm$ and an initial volume $V_i = 250\ cm^3$. The volume of the piston is increased by pulling out the lid until the final volume is $V_f = 3.70\, V_i$. At the same time the volume is increased, the temperature is decreased by 50.0 K with an external temperature controller. What is the final pressure p_f *in atmospheres*? Give the answer in the form “X atm”, where X is the number of atmosphere.

T_i = Given
P_i = Given
V_i = Given
V_f = Given
T_f = Given
P_f = ?

Using a calculation from problem (6),

$$\frac{P_i V_i}{P_f V_f} = \frac{n_i R T_i}{n_f R T_f}$$

The number of particles does not change and R is a constant,

$$\begin{aligned} P_i V_i T_f &= P_f V_f T_i \\ \frac{P_i V_i T_f}{V_f T_i} &= P_f \end{aligned}$$

8) A mass $m_{Pb} = 4.00\ kg$ of lead shot at a temperature $T_i = 97.9°C$ is poured into $m_w = 4.00\ kg$ of water at $T_w = 20.5°C$. Find the final temperature T_f of the mixture. Give the answer in degrees Celsius, using the unit "C". (CAPA can't read a superscripted degree zero.) Use $c_{water} = 4187\ \frac{J}{kg°C}$ and $c_{lead} = 128\ \frac{J}{kg°C}$.

m_{Pb} = Given
T_i = Given
m_w = Given
T_w = Given
$T_f = ?$

We can use the equation $\Delta Q = mc\Delta T$ to relate the heat gained by the water and lost by the lead.

$$-m_{Pb}c_{lead}\Delta T_{lead} = m_{water}c_{water}\Delta T_w$$

So,

$$\begin{aligned} m_{water}c_{water}(T_f - T_w) + m_{Pb}c_{lead}(T_f - T_i) &= 0 \\ m_{water}c_{water}T_f + m_{Pb}c_{lead}T_f &= m_{water}c_{water}T_w + m_{Pb}c_{lead}T_i \\ T_f &= \frac{m_{water}c_{water}T_w + m_{Pb}c_{lead}T_i}{m_{water}c_{water} + m_{Pb}c_{lead}} \end{aligned}$$

9) A brass cube has an edge length of $L = 35.0\ cm$. What is the increase in its surface area ΔA when it is heated from $T_i = 21.0°C$ to $T_f = 82.0°C$? Data: $\alpha_{Brass} = 19 \times 10^{-6}\ °C^{-1}$ (at $20°C$).

L = Given in centimeters
T_i = Given
T_f = Given
α_{Brass} = Given
$\Delta A = ?$

The initial surface area can easily be calculated.

$$A_i = 6L^2$$

To find the new length of each side (and the new surface area), use the equation

$$\begin{aligned} \alpha &= \frac{\Delta L}{L\Delta T} \\ \alpha L\Delta T &= \Delta L \end{aligned}$$

Then find the difference in surface area

$$A_f - A_i = 6(L + \Delta L)^2 - 6L^2$$

Remember to convert L to meters. The area difference will be in square meters.

10) At $T_i = 20°C$, an iron bar has length $L_{Fe} = 1.5000\ m$ long and a brass bar has length $L_{brass} = 1.4965\ m$ long. Find the temperature T at which they will have the same length. Data: $\alpha_{Iron} = 12 \times 10^{-6}\ °C^{-1}$, $\alpha_{Brass} = 19 \times 10^{-6}\ °C^{-1}$ (at $20°C$).

$T_i =$ Given
$L_{Fe} =$ Given
$L_{brass} =$ Given
$\alpha_{Iron} =$ Given
$\alpha_{brass} =$ Given
$T_f = ?$

We want to know when the two rods have equal length. This will happen when

$$L_{brass} + \Delta L_{brass} = L_{Fe} + \Delta L_{Iron}$$

We solved for ΔL in the previous problem. Substituting in,

$$\begin{aligned}
L_{brass} + \alpha_{brass} L_{brass} \Delta T &= L_{Fe} + \alpha_{Iron} L_{Fe} \Delta T \\
\alpha_{brass} L_{brass} \Delta T - \alpha_{Iron} L_{Fe} \Delta T &= L_{Fe} - L_{brass} \\
\Delta T \left(\alpha_{brass} L_{brass} - \alpha_{Iron} L_{Fe} \right) &= L_{Fe} - L_{brass} \\
\Delta T = T_f - T_i &= \frac{L_{Fe} - L_{brass}}{\left(\alpha_{brass} L_{brass} - \alpha_{Iron} L_{Fe} \right)} \\
T_f &= \frac{L_{Fe} - L_{brass}}{\left(\alpha_{brass} L_{brass} - \alpha_{Iron} L_{Fe} \right)} + T_i
\end{aligned}$$

11) Choose all correct statements, e.g. B, AC, BCD, etc. NOTE!!! You may need to know the coefficient of expansion of copper, which is NOT listed in the text. (slightly less than brass).

QUESTION:
A) When the Celsius temperature doubles, the Fahrenheit temperature doubles.
B) A copper plate has a hole cut in its center. When the plate is heated, the hole gets smaller.
C) When heat is added to a system, the temperature must rise.
D) When the pendulum of a grandfather clock is heated, the clock runs more slowly.
E) Cold objects do not radiate heat energy.
F) In a bimetallic strip of aluminum and brass which curls when heated, the aluminum is on the inside of the curve.

ANSWER:
A) FALSE: A Fahrenheit degree is not the same size as a Celsius degree.
B) FALSE: When heated, all linear dimensions expand (the hole gets larger).
C) FALSE: could use up heat.

D) TRUE: The pendulum will increase in length, increasing the moment of inertia, and slowing the angular movement.
E) FALSE: All objects with an absolute temperature above absolute zero (all objects) radiate heat energy.
F) FALSE: Aluminum has a higher expansion coefficient than does brass so it will expand further. It will be on the outside of the curve.
NOTE: CAPA is looking for the letters of the correct answers. In this case, D.

12) How much energy is lost (that is, what is the decrease in energy $-\Delta E$) when $m = 136.0\ g$ of steam, at a temperature $T_i = 181.0\ °C$, is cooled and frozen into $m = 136.0\ g$ of ice at $T_f = 0\ °C$? take the specific heat of steam to be $c_{steam} = 2.01\ kJ/kg \cdot K$, the heat of vaporization of water is $L_v = 539\ cal/g$, the heat of fusion of ice is $L_F = 79.7\ cal/g$. use the known value of the specific heat of water c_{water}.

$-\Delta E = ?$
$m_{steam} = m_{ice} = m_{water} = m =$ Given in grams.
$T_i =$ Given
$m_{ice} =$ Given
$T_f =$ Given
$c_{steam} =$ Given
$L_v =$ Given in cal/g
$L_f =$ Given in cal/g
$c_{water} = 4.184\ \frac{kJ}{kg \cdot K}$

This problem has four parts. Cool the steam to $100°C$, condense into water, cool the water to $0°C$, and freeze the water.

$$\begin{aligned}\Delta Q_1 &= mc_{steam}\Delta T\\ \Delta Q_2 &= mL_v\\ \Delta Q_3 &= mc_{water}\Delta T\\ \Delta Q_4 &= mL_f\end{aligned}$$

ΔQ_1 is the energy that must be removed from the steam in order to bring it to $100°C$. ΔQ_2 is the energy that must be removed to condense the steam into water. ΔQ_3 is the energy that must be removed to cool the water to $0°C$. ΔQ_4 is the energy that must be removed to freeze the water. In all, the total energy that is removed from the system,

$$Q_{tot} = \Delta Q_1 + \Delta Q_2 + \Delta Q_3 + \Delta Q_4$$

Before calculating anything, make sure to convert your mass to kilograms as well as L_f and L_v to kJ/kg since that is what c_{steam} and c_{water} are given in. You could also convert them the other way around. To do this, use the conversion factors $\frac{1\ Cal}{1000\ cal}$, $\frac{4184\ J}{1\ Cal}$, $\frac{1\ kJ}{1000\ J}$ and, of course, $\frac{1000\ g}{1\ kg}$. You do not need to convert the temperature to Kelvin since you are concerned about the *change* in temperature. The change in Celsius temperature is the same magnitude as the change in Kelvin. If you converted L_f and L_v then your answer will have units of kJ.

13) It is possible to warm your hands on a cold day by rubbing them together. Assume that the coefficient of sliding friction between your hands is 0.5 and that the normal force between the hands is 44.0 N, and that you rub them together with an average speed of 33.0 cm/s. Assume that the mass of each of your hands is 323.0 g, that the specific heat of your hands is 4.2 $kJ/kg \cdot K$, and that all the heat generated goes into raising the temperature of your hands. How long must you rub your hands together to produce a 6.0 $°C$ increase in temperature?

$\mu =$ Given (coefficient of sliding friction)
$F_N =$ Given (Normal force)
$s =$ Given (speed) in cm/s
$m =$ Given in grams
$c_{hands} =$ Given
$\Delta T =$ Given
$t = ?$

This is also a multipart problem. First, calculate the amount of heat that must be generated to raise the temperature of your hands by ΔT.

$$\Delta Q = 2mc_{hands}\Delta T$$

Where the 2 is introduced because you have two hands. To find the time necessary for your hands to heat up by this amount, we must relate the work you must do to heat your hands to the time it takes to heat them. This is precisely the definition of power.

$$P = \frac{W}{t}$$

But another, obscure formula for power is that

$$P = Fv$$

By having two formulae for power, you can set them equal and solve for the unknown (time in this case).

$$\begin{aligned} Fv &= \frac{W}{t} \\ t &= \frac{W}{Fv} \qquad (1) \end{aligned}$$

Where

$$F = \mu F_N$$

is the force of friction,

$$v = s$$

is the speed at which you rub your hands together (convert to m/s), and

$$W = \Delta Q = 2mc_{hands}\Delta T$$

is the work which must be done to heat your hands (convert mass to kg). Another, less obvious, step is to convert c_{hands} to J (since it is in kJ). Remember that $\frac{1000\ J}{1\ kJ}$. Looking at the formula for time below, we will end up with

$$\frac{kg * \frac{kJ}{kgK} * K}{N * m/s}.$$

A joule is defined to be a Newton * meter. But the units will not cancel if the value of c_{hands} is left in kJ. Substituting this back in to equation (1),

$$t = \frac{2mc_{hands}\Delta T}{\mu F_N * s}$$