Solution Derivations for Capa #13

1) Identify the following waves as T-Transverse, or L-Longitudinal. If the first is T and the rets L, enter TLLL).

QUESTION:
A) The 'WAVE' made by fans at sports events.
B) Wave on a slinky toy, snugly confined in a long narrow tube.
C) Waves in a guitar string that is plucked.
D) Waves on the surface of a bass drum.
E) Band music at a football game.

ANSWER:
A) T, the fans are only moving up and down, not with the wave.
B) L, the slinky cannot move up and down due to its confinement.
C) T, the guitar string is vibrating back and forth perpendicular to the axis.
D) T, on the surface, the material covering the drum is moving perpendicular to the drum surface. This action compresses the air which makes the sound.
E) L, the music we hear is from the air being compressed.

2) The transverse displacement of a stretched string from equilibrium as a function of time and position is given by:

$$y = 0.13\cos(9x - 99t).$$

x and y are in m; t is in s. (Select T-True, F-False, G-Greater than, L-Less than, E-Equal to, If the first is T and the rest F, enter TFFF).

The general equation of the wave:

$$y\left(x,t\right) = A\sin\left(kx - \omega t\right)$$

QUESTION:
A) The wave travels in the positive x direction.
B) The speed of the wave is ..... ## m/s.
C) The period is ..... ## seconds.
D) The wavelength is ..... ## m.

ANSWER:
A) The wave travels in the positive x direction (to the right) if its formula is the form $f\left(x - vt\right)$. It will travel in the negative x direction (to the left) if the formula is $f\left(x + vt\right)$.
B) The speed of the wave can be calculated from $v = \frac{\omega}{k}$.
C) $\omega = \frac{2\pi}{T}$, so $T = \frac{2\pi}{\omega}$.
D) $k = \frac{2\pi}{\lambda}$, so $\lambda = \frac{2\pi}{k}$.

3) A typical steel B-string in a guitar resonates in its fundamental frequency at 240 $Hertz$. The length of the string is 0.660 m. What is the wave velocity along the string?

f = Given
L = Given

To find the wavelength associated with the fundamental frequency, use the equation

$$\lambda_n = \frac{2L}{n}$$

where $n = 1$ for the fundamental frequency. This equation is what determines a standing wave. Any integer number of half wavelengths determines a standing wave since every half wavelength is a node in the wave. Whatever integer n is used determines the harmonic level, where $n = 1$ is the fundamental frequency (1st harmonic). The velocity is then calculated from $v = \lambda f$.

4) John is listening to a horn. He knows the frequency of the horn is 300 Hz when both he and the horn are at rest. If he hears a pitch of 330 Hz, there are clearly several possibilities. (Give ALL correct answers, i.e., B, AC, BCD...)

QUESTION:
A) John is moving towards the horn at rest.
B) Both can be moving and have the same speed.
C) Both can be moving and have different speeds.
D) Both can be moving, in opposite directions.
E) The distance between John and the horn is increasing with time.
F) Both cannot be moving in the same direction.

ANSWER:
A) True, when John moves towards the horn, it is the same as if he stood still and the horn moved towards him.
B) True, if they are moving toward each other at the right speed, John would hear more waves per unit time, thus a higher frequency.
C) True, the horn could be getting closer to John.
D) True, same as B, they can approach each other to achieve this effect.
E) False, it must be decreasing to hear a higher frequency.
F) False, they could be. If both are moving and the horn is catching up to John, this effect could be achieved.
Be careful, this problem is looking for the TRUE answers... so answer ABCD (or whatever answers are true).

5) A 60 year old person has a threshold of hearing at $f = 10,000$ Hz of 75.0 dB. By what factor must the intensity of a sound wave of that frequency, audible to a typical young adult (sound level = 43.0 dB), be increased so that it is heard by the older person.

f = Given
β_{old} = Given
β_{young} = Given

In order for the older person to hear this sound, the intensity must be increased above the threshold of hearing. That is, β must be increased. The problem is asking for what factor intensity must increase.

$$\beta_{young} = 10\log\left(\frac{I_i}{I_0}\right)$$
$$\beta_{old} = 10\log\left(\frac{I_f}{I_0}\right)$$

where I_i and I_f are the initial and final intensities. We can solve each equation for I_i or I_f.

$$\frac{\beta_{young}}{10} = \log\left(\frac{I_i}{I_0}\right)$$
$$10^{\frac{\beta_{young}}{10}} = \frac{I_i}{I_0}$$
$$I_0 10^{\frac{\beta_{young}}{10}} = I_i$$

and

$$\frac{\beta_{old}}{10} = \log\left(\frac{I_f}{I_0}\right)$$
$$10^{\frac{\beta_{old}}{10}} = \frac{I_f}{I_0}$$
$$I_0 10^{\frac{\beta_{old}}{10}} = I_f$$

where

$$I_0 = 10^{-12}\frac{W}{m^2}$$

is the threshold of human hearing. To find the factor the intensity must increase, think that some number x must be multiplied by I_i to get I_f. So,

$$x * I_i = I_f$$
$$x = \frac{I_f}{I_i}$$

This ratio is unitless.

6) A certain loudspeaker has a diameter of 26 cm. At what frequency will the wavelength of the sound it emits in air be equal to its diameter? (*Note:* If the wavelength is large compared to the diameter of the speaker, then the sound waves spread out almost uniformly in all directions from the speaker; but when the wavelength is small compared to the diameter of the speaker, the wave energy is propagated mostly in the forward direction.) The SI unit of frequency is “Hz”.

$d =$ Given in cm.

To find the frequency at which the wavelength equals the diameter, simply note that $\lambda = d$. The general equation relating frequency and wavelength is $v = \lambda f$. This is a constant for any medium. To find the frequency,

$$f = \frac{v}{\lambda}$$

Since we are concerned with the sound in the air, the velocity in air is 343 m/s. Remember to perform the conversion from cm to m for the diameter in this problem. The answer will have units of s^{-1} or Hz.

7) The noise level in an empty examination hall is 41 dB. When 100 physics students are writing an exam, the sounds of heavy breathing and pens traveling rapidly over paper cause the noise level to rise to 57 dB (not counting the occasional groans). Assuming that each student contributes and equal amount of noise power, find the noise level when half of the students have left the examination hall.

$\beta_i =$ Given
$\beta_f =$ Given
100 students

We must find the sound intensity when the room is empty and when it is full. Then we can determine how much each student contributes to that intensity.

$$\beta_i = 10 \log \left(\frac{I_i}{I_0} \right)$$

where

$$I_0 = 10^{-12}.$$

Looking back to problem 5, the intensity I_i is

$$I_i = I_0 10^{\frac{\beta_i}{10}}$$

Similarly, the final intensity I_f is

$$I_f = I_0 10^{\frac{\beta_f}{10}}$$

The change in intensity is $\Delta I = I_f - I_i$. Each student contributes an amount of $\frac{\Delta I}{100}$ to the change in intensity. If half the students leave, we can multiply the intensity each student creates by 50 to find the intensity created by half the students. Then we can find the decibel rating corresponding to that intensity. So,

$$I_{half} = 50 * \frac{\Delta I}{100}$$

$$
\begin{aligned}
&= \frac{1}{2}\left(I_f - I_i\right) \\
&= \frac{1}{2}\left(I_0 10^{\frac{\beta_f}{10}} - I_0 10^{\frac{\beta_i}{10}}\right) \\
&= \frac{1}{2} I_0 \left(10^{\frac{\beta_f}{10}} - 10^{\frac{\beta_i}{10}}\right) \\
\beta_{half} &= 10 \log\left(\frac{I_{half}}{I_0}\right) \\
&= 10 \log\left(\frac{\frac{1}{2} I_0 \left(10^{\frac{\beta_f}{10}} - 10^{\frac{\beta_i}{10}}\right)}{I_0}\right) \\
&= 10 \log\left(\frac{1}{2}\left(10^{\frac{\beta_f}{10}} - 10^{\frac{\beta_i}{10}}\right)\right)
\end{aligned}
$$

8) The pilot of a light aircraft wishes to reduce significantly engine noise. If he wears a head set which reduces the noise level by $\Delta\beta = -26.1\ dB$, by what factor f is the intensity of the noise attenuated? (The factor f is defined this way: if I_0 is the original noise level, and I is the attenuated noise level, then $I = f\ I_0$.)

First of all, attenuate means to lessen in severity, value, intensity, etc.
$I = f * I_0$ Given
$\Delta\beta =$ Given

So

$$f = \frac{I}{I_0} = \frac{I_f}{I_i}$$

The change of I to I_f and of I_0 to I_i was done to avoid confusion with I_0, the threshold of human hearing which equals 10^{-12}. We need to find the initial and the final intensity. From problem 5, the initial intensity I_i is

$$I_i = I_0 10^{\frac{\beta_i}{10}}$$

Similarly, the final intensity I_f is

$$I_f = I_0 10^{\frac{\beta_f}{10}}$$

Since we are looking at the change in intensity, it doesn't matter for what initial decibel intensity we started with as long as the change is the same. So we can choose β_i to be anything and $\beta_f - \beta_i = \Delta\beta$, or $\beta_f = \beta_i + \Delta\beta$. If $\Delta\beta$ is negative, use it in the formula as such. Pick 100 if you are at a loss for β_i (or 50 if your calculator cannot handle large numbers). Then calculate I_i and I_f and then the ratio $\frac{I_f}{I_i}$ to get f. The answer is unitless.

9) Hovering over the pit of hell, Satan observes that, although a stationary physics student shrieks at frequency $f_0 = 905.4\ Hz$, when the student falls aways into the nether regions, the pitch of his shriek is $f_1 = 883.4\ Hz$. What is the speed v of descent of the physics student? Assume that the velocity of sound is $v_{sound} = 340.0\ m/s$.

f_0 = Given (frequency at rest)
f_1 = Given (frequency when moving)
v_{sound} = Given
$v_{student} = u = ?$

The Doppler effect says that the frequency of a moving object (f_1) is

$$f_1 = \frac{f_0}{1 - \frac{u}{v}}$$

where f_0 is the frequency of the stationary object, v is the wave speed (in this case, the speed of sound v_{sound}), u is the speed of the source with respect to the medium (in this case, the falling rate of the student $v_{student}$), and the minus sign in the denominator indicates that the source is moving away from the observer. To solve for u,

$$\begin{aligned} \left(1 - \frac{u}{v}\right) f_1 &= f_0 \\ 1 - \frac{u}{v} &= \frac{f_0}{f_1} \\ 1 - \frac{f_0}{f_1} &= \frac{u}{v} \\ u &= v\left(1 - \frac{f_0}{f_1}\right) \end{aligned}$$

Using more symbolic names,

$$v_{student} = v_{sound}\left(1 - \frac{f_0}{f_1}\right)$$

The units of velocity are m/s and CAPA is looking for the speed, so take the absolute value of your result.

10) A listener is a distance $r = 20.0\ m$ from a small explosion and reports a sound of intensity 72.0 dB. What is the distance R from the explosion at which listeners hear an intensity of 52.0 dB?

r = Given
β_i = Given
β_f = Given
$R = ?$

We are given the intensity in decibels, but we can find the intensity I in W/m^2 (power/area) and proceed from there. Since the power of the loud speaker does not change, it will connect the first and second parts. Again, from problem 5, the initial intensity I_i is

$$I_i = I_0 10^{\frac{\beta_i}{10}}$$

Similarly, the final intensity I_f is

$$\begin{aligned} I_f &= I_0 10^{\frac{\beta_f}{10}} \\ I_i &= \frac{P}{A} \end{aligned}$$

Sound waves are emitted spherically, so the area is $\frac{4}{3}\pi r^2$. This allows us to solve for the power.

$$\begin{aligned} P &= A * I_i \\ P &= \frac{4}{3}\pi r^2 I_i \end{aligned}$$

Similarly,

$$\begin{aligned} \frac{P}{A} &= I_f \\ \frac{P}{\frac{4}{3}\pi R^2} &= I_f \end{aligned}$$

Solving for R,

$$\begin{aligned} P &= I_f \frac{4}{3}\pi R^2 \\ \frac{3}{4}\frac{P}{I_f \pi} &= R^2 \\ R &= \sqrt{\frac{3}{4}\frac{P}{I_f \pi}} \end{aligned}$$

But

$$P = \frac{4}{3}\pi r^2 * I_i$$

so

$$R = \sqrt{\frac{3}{4}\frac{\frac{4}{3}\pi r^2 I_i}{I_f * \pi}} = \sqrt{\frac{r^2 I_i}{I_f}}$$