

Solution Derivations for Capa #11

1) A horizontal circular platform ($M = 128.1 \text{ kg}$, $r = 3.11 \text{ m}$) rotates about a frictionless vertical axle. A student ($m = 68.3 \text{ kg}$) walks slowly from the rim of the platform toward the center. The angular velocity ω of the system is 2.1 rad/s when the student is at the rim. Find ω when the student is 2.23 m from the center. Use “rad/s” as units.

M of platform = Given

R of platform = Given

m of student = Given

r_i of student = Given

r_f of student = Given

ω_i = Given

ω_f = ?

This is a simple conservation of angular momentum problem.

$$\begin{aligned}L_i &= L_f \\I_i\omega_i &= I_f\omega_f\end{aligned}$$

The moments of inertia are just the sum of moments of inertia of both the disk and the student.

$$\begin{aligned}I_i &= \frac{1}{2}MR^2 + mr_i^2 \\I_f &= \frac{1}{2}MR^2 + mr_f^2\end{aligned}$$

Realizing that the student starts at the rim of the platform, the initial radius of the student is equal to the radius of the platform, or $r_i = R$. Plugging these new values in and solving for ω_f ,

$$\begin{aligned}\left(\frac{1}{2}MR^2 + mR^2\right)\omega_i &= \left(\frac{1}{2}MR^2 + mr_f^2\right)\omega_f \\ \frac{\left(\frac{1}{2}MR^2 + mR^2\right)\omega_i}{\left(\frac{1}{2}MR^2 + mr_f^2\right)} &= \omega_f\end{aligned}$$

No conversions should be necessary since ω_i is given in rad/s and you are asked to answer in rad/s .

2) Two particles, each with mass $m = 5.8 \text{ g}$, are fastened to each other and to a rotation axis as point P , by two thin rods, each with length $L = 0.65 \text{ m}$ and a mass of 8.1 g , as shown. The combination rotates around the rotation axis with an angular velocity of 13.6 rad/s . Find the rotational inertia of the combination about P ?

m = Given in grams
 L = Given
 M = Given in grams
 ω = Given
 I = ?

The moment of inertia of the system about the axis P is the sum of the moments of inertia for each rod and mass. Since the rods are equal length and mass and since they are attached in a line, they can be treated as a single rod of mass $2M$ and length $2L$.

$$I = \frac{1}{3} (2M) (2L)^2 + mr_1^2 + mr_2^2$$

Since the masses are given in grams, first convert them to kilograms by dividing by 1000. The radius to the first mass is the length of the first rod. The radius of the second mass is the length of both rods.

$$I = \frac{8}{3} ML^2 + mL^2 + m(2L)^2$$

Units for moment of inertia are $kg \cdot m^2$.

3) A large plate is balanced at its center and two students of equal mass stand at its center. The plate is rotated on a frictionless pivot about an axis through its center and perpendicular to its face. The students then begin to walk out towards opposite edges. (Select T-True, F-False, I-Increases, D-Decreases, S-Stays the same.).

QUESTION:

- A) The rate of rotation ... as the students walk outward.
- B) The students produce a net torque on the plate.
- C) When the students reach the edge and stop, the plate will have the same angular speed as when they started.
- D) The students do work in walking outward
- E) The total angular momentum of the system ... as the students walk outward.

ANSWER:

- A) D. As the mass of the students gets further away from the axis of rotation, the rate slows down (remember the skater with arms out).
- B) T. No explanation currently...
- C) F. The plate slows down as they get further out. The rotation rate does not matter if they are moving or not.
- D) T. The students exert a force on the plate (and the plate on the students) when walking out. Thus, that force over the distance walked causes work to be done. Consequently the force of the plate on the students is what slows the rotation rate.
- E) S. Angular momentum is conserved.

4) In the Figure, a block m_1 sits on a table. There *is* static friction between the block and table. Block m_2 hangs from a knot, as shown. Call the tension in the rope connecting the knot to m_1 , “T1”. Call the tension in the rope connecting the knot to m_2 , “T2”. Call the tension in the third rope (the one tipped up by an angle θ , connecting the knot to the wall, “T3”. *The system is in equilibrium.* Which of the following statements are true? (If A and E are true, and the others are not, enter TFFFT)

QUESTION:

- A) T3 is equal in magnitude to T1.
- B) The net force on the knot is zero.
- C) If the table was completely frictionless, the system as shown could not be in equilibrium.
- D) The tension T2 must equal m_2g .
- E) The force of static friction on m_1 equals T1.

ANSWER:

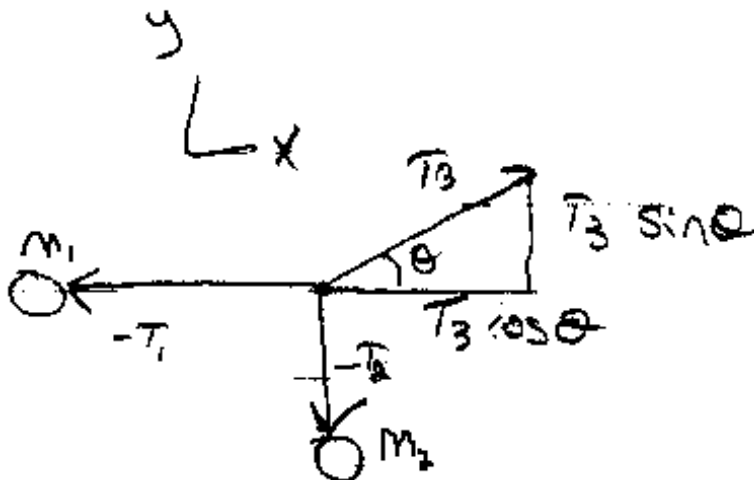
- A) F. Since T3 is directed an and angle θ above the horizontal, it is not equal. T3’s horizontal component, is, however.
- B) T. Since the system is in equilibrium, all the forces sum to 0.
- C) T. If there was no horizontal force (from friction) opposing T3’s horizontal component, the system would not be in equilibrium.
- D) F. Since the mass is the only object connected to the string, it must equal the weight.
- E) T. Again, since the mass is the only object connected to the string, it must equal to the force of friction.

5) In the previous problem, block m_1 weighs 709 N. The coefficient of static friction between the block and the table is 0.27 and the angle θ is 34.3° . Find the maximum weight of block m_2 for which block m_1 will remain at rest. (Answer ins Newtons, N)

$m_1 =$ Given
 $\mu_s =$ Given
 $\theta =$ Given

From a free body diagram, we can see that

$$\begin{aligned} \sum F_y &= -T_2 + T_3 \sin \theta \\ T_3 \sin \theta &= m_2 g \\ T_3 &= \frac{m_2 g}{\sin \theta} \\ \sum F_x &= T_3 \cos \theta - T_1 \\ T_1 &= T_3 \cos \theta = \frac{m_2 g}{\sin \theta} \cos \theta \\ m_2 &= \frac{T_1 \sin \theta}{g \cos \theta} = \frac{T_1}{g} \tan \theta \end{aligned}$$



Since T_1 is due to friction,

$$T_1 = \mu_s N = \mu_s m_1 g$$

$$m_2 = \frac{\mu_s m_1 g}{g} \tan \theta = \mu_s m_1 \tan \theta$$

6) A physics student is sitting on a frictionless platform that is rotating with an angular speed of $\omega_i = 3.8 \text{ rad/s}$. His arms are outstretched, and he is holding a heavy weight in each hand. The moment of inertia of the student, the extended weights, and the platform about the rotation axis is $I_i = 8.5 \text{ kg} \cdot \text{m}^2$. When the student pulls the weights inward toward his body, the moment of inertia decreases to $I_f = 5.0 \text{ kg} \cdot \text{m}^2$. What is the resulting angular speed, ω_f , of the platform?

$\omega_i = \text{Given}$
 $I_i = \text{Given}$
 $I_f = \text{Given}$
 $\omega_f = ?$

This is another conservation of angular momentum problem.

$$L_i = L_f$$

$$I_i \omega_i = I_f \omega_f$$

$$\omega_f = \frac{I_i \omega_i}{I_f}$$

This will give you an answer in rad/s.

7) By how much is the kinetic energy of the system increased? (Think about where this increase in energy comes from!)

Kinetic energy is related to the moment of inertia and the angular speed.

$$KE = \frac{1}{2}I\omega^2$$

To find the increase in kinetic energy, subtract the initial from the final.

$$\Delta KE = \frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}(I_f\omega_f^2 - I_i\omega_i^2)$$

8) Compact discs and long-playing records are made from similar materials. The former have a diameter of about 12 *cm*, and the latter, about 32 *cm*. When in use, records spin at $33\frac{1}{3}$ *rev/min*, and compact discs spin at, say, 365 *rev/min*. Ignoring the holes in both objects and assuming that a compact disc has half the thickness of a record and 0.90 of its density, what is the ratio of the angular momentum of a compact disc in use to that of a record?

R_{CD} = Given

R_{LP} = Given

ω_{LP} = Given

ω_{CD} = Given

h_{CD} = Thickness of the CD = Given in terms of the record (LP). Assume it is a coefficient, y , times h_{LP} . That is, if your CD height is half of your record height, $y = \frac{1}{2}$.

d_{CD} = Given as a coefficient times d_{LP} . Assume it is a coefficient, x , times d_{LP} . That is, if your CD density is 90% of your record density, $x = 0.9$.

This problem deals with angular momentum.

$$L = I\omega$$

In this case, you are asked to ignore the holes in the CD and LP, so the object is just like a solid disk. The moment of inertia for a disk is given by $\frac{1}{2}mR^2$. So,

$$L = \frac{1}{2}mR^2\omega$$

You are asked to find the ratio of the angular of momentum of the CD to that of a record

$$\frac{L_{CD}}{L_{LP}} = \frac{\frac{1}{2}m_{CD}R_{CD}^2\omega_{CD}}{\frac{1}{2}m_{LP}R_{LP}^2\omega_{LP}} = \frac{m_{CD}R_{CD}^2\omega_{CD}}{m_{LP}R_{LP}^2\omega_{LP}}$$

We know everything but the mass of the CD and record. Density is mass/volume. So mass = density * volume. To find the volume of a CD, use the formula for a right cylinder (i.e. πR^2h). The h in this case will be $y * h_{LP}$. So the volume of the CD is $\pi R_{CD}^2 * y * h_{LP}$. To find the mass of the CD, multiply the density

given by the volume. The given density is in terms of the density of the record ($x * d_{LP}$). So, the mass of the CD is just

$$m_{CD} = v * d$$

$$m_{CD} = \pi R_{CD}^2 * y * h_{LP} * x * d_{LP}$$

The mass of the record can be found also by finding its volume ($\pi R_{LP}^2 h_{LP}$) and its density (d_{LP}). Don't worry that some of these terms are unknown because they will cancel in the end. So,

$$m_{LP} = v * d$$

$$m_{LP} = \pi R_{LP}^2 h_{LP} * d_{LP}$$

Plugging these back into the ratio of angular momenta,

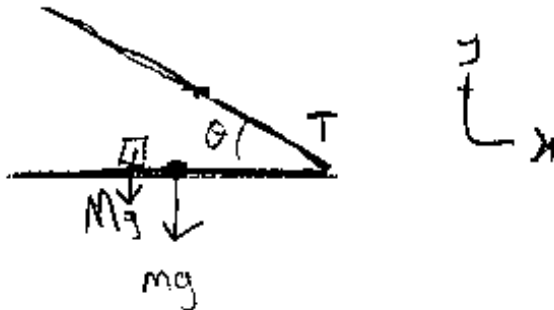
$$\frac{L_{CD}}{L_{LP}} = \frac{m_{CD} R_{CD}^2 \omega_{CD}}{m_{LP} R_{LP}^2 \omega_{LP}} = \frac{\pi R_{CD}^2 * y * h_{LP} * x * d_{LP} * R_{CD}^2 \omega_{CD}}{\pi R_{LP}^2 h_{LP} * d_{LP} * R_{LP}^2 \omega_{LP}}$$

Look what cancels. π , h_{LP} , and d_{LP} . Simplifying,

$$\frac{L_{CD}}{L_{LP}} = \frac{R_{CD}^2 * y * x * R_{CD}^2 \omega_{CD}}{R_{LP}^2 * R_{LP}^2 \omega_{LP}} = \frac{R_{CD}^4 * y * x * \omega_{CD}}{R_{LP}^4 * \omega_{LP}}$$

This is your answer, and ratio's have no units.

9) A uniform horizontal bar of length $L = 2 \text{ m}$ and weight 219 N is pinned to a vertical wall and supported by a thin wire that makes an angle of $\theta = 24^\circ$ with the horizontal. A mass M , with a weight of 311 N , can be moved anywhere along the bar. The wire can withstand a maximum tension of 515 N . What is the maximum possible distance from the wall at which mass M can be placed before the wire breaks?



$L = \text{Given}$

$mg = \text{Given (weight of rod)}$

$\theta = \text{Given}$

$Mg = \text{Given (weight of mass)}$

$T_{\max} = \text{Given}$
 $r = ?$ (radius to mass from axis A)

This system is obviously in equilibrium. In order for a system to be in equilibrium, the sum of forces must be zero and the sum of torques must be zero. However, we only need to be concerned with the torques for this part of the problem. Looking at a free-body diagram, we can find the torques about the axis A . The sum of torques is equal to the torque of the wire, the torque of the rod, and the torque of the mass. Remember $\tau = rF \sin \theta$

$$\tau = TL \sin \theta - \frac{1}{2}Lmg - rMg = 0$$

Solving for r ,

$$TL \sin \theta - \frac{1}{2}Lmg = rMg$$
$$\frac{TL \sin \theta - \frac{1}{2}Lmg}{Mg} = r$$

10) With M placed at this maximum distance what is the horizontal component of the force exerted on the bar by the pin at A ?

To find the horizontal component of the force, we can again look at the free-body diagram. The only forces acting in the horizontal direction are the component of the tension, and the force of the wall on the rod at point A . So, they must be equal.

$$F_x = T \cos \theta$$

11) Misako is about to do a push-up. Her center of gravity lies directly above a point on the floor which is $d_1 = 1.04 \text{ m}$ from her feet and $d_2 = 0.62 \text{ m}$ from her hands. If her mass is 55.0 kg , what is the force exerted by the floor on her hands?

$d_1 = \text{Given}$
 $d_2 = \text{Given}$
 $m = \text{Given}$

This problem involves torque once again. The person is pushing on the floor in two different spots. She is exerting a torque on each spot on the ground. Since she nor the ground is moving, the torque must be equal. In most cases, the center of gravity will also be the center of mass. Call the force of her hands (a distance of d_2 from the center of gravity) F_2 . Call the force of her feet (a distance of d_1 from her center of gravity) F_1 . Since she is pushing at right angles on the ground, and the force of her hands is exerting an opposite torque due to the force of her feet so the total torque can be calculated by

$$\tau = F_1 d_1 - F_2 d_2 = 0$$

However, we must find F_1 and F_2 now. Realizing that her mass is m , she is only exerting a force of mg on the ground. Thus,

$$F_1 + F_2 = mg$$

Solving for F_1 in this equation will give F_2 in the second equation (the force of her hands on the ground). This is also the force of the ground on her hands since the ground pushes back with an equal and opposite force.

$$F_1 = mg - F_2$$

Plugging back into the torque equation,

$$\begin{aligned} 0 &= d_1(mg - F_2) - F_2d_2 \\ 0 &= d_1mg - d_1F_2 - F_2d_2 \\ d_1F_2 + d_2F_2 &= d_1mg \\ F_2(d_1 + d_2) &= d_1mg \\ F_2 &= \frac{d_1mg}{d_1 + d_2} \end{aligned}$$

Force has units of Newtons (N).

12) What is the force exerted by the floor on her feet?

This problem is simply an extension of the last. Since we know F_2 , you can use the equation $F_1 + F_2 = mg$ to find the other force (the force of the ground on her feet). So,

$$F_1 = mg - F_2$$

13) The weightless horizontal bar in the figure below is in equilibrium. Scale B reads 4.40 kg. The distances in the figure (which is not to scale) are: $D_1 = 6.0 \text{ cm}$, $D_2 = 7.5 \text{ cm}$, and $D_3 = 6.0 \text{ cm}$. The mass of block X is 0.98 kg and the mass of block Y is 2.04 kg. Determine what the reading on scale A must be.

$B = \text{Given}$
 $D_1 = \text{Given}$
 $D_2 = \text{Given}$
 $D_3 = \text{Given}$
 $m_x = \text{Given}$
 $m_y = \text{Given}$
 $A = ?$

This problem involves the torque. The torque on A must equal the torque on B since the system is not moving. The torque on B can be found by summing the torque from mass x and mass y . Since mass z is on the axis of rotation (at B), it produces no torque.

$$\tau_B = (D_2 + D_3)m_x + D_3m_y$$

The torque on A can be calculated by referencing it from B . In other words, what torque must be applied at A to counteract the torques from m_x and m_y ? This can be found by denoting the variable m_a to represent the mass (the scale reading) at A to counteract the torque calculated above. That is,

$$m_a(D_1 + D_2 + D_3) = (D_2 + D_3)m_x + D_3m_y$$

Thus, the scale reading at A (that is, m_a)

$$m_a = \frac{(D_2 + D_3)m_x + D_3m_y}{D_1 + D_2 + D_3}$$