Solution Derivations for Capa #10

1) The flywheel of a steam engine runs with a constant angular speed of 172 rev/min. When steam is shut off, the friction of the bearings and the air brings the wheel to rest in 1.0 hours. What is the magnitude of the constant angular acceleration of the wheel? (Answer in rev/min^2)

ω_0 = Given
t = Given in hours, convert to minutes.

Since ω is given in $\frac{rev}{min}$, we can directly substitute into the angular kinematics equations. For this problem, $\omega = \omega_0 + \alpha t$ comes in handy. Solving for α,

$$\alpha = \frac{\omega - \omega_0}{t}$$

In this case, ω (the final rotational speed) is zero since the engine flywheel stops. So,

$$\alpha = \frac{-\omega_0}{t}.$$

Units are $\frac{rev}{min^2}$ and CAPA is looking for the magnitude of the answer.

2) How many rotations does the wheel make before coming to rest? (No units required)

For this problem, remember from translational kinematics that $x = \bar{v}t$. Similarly, in rotational kinematics, $\theta = \bar{\omega}t$. Average angular speed is given by $\frac{1}{2}(\omega + \omega_0)$. Thus, the equation becomes

$$\theta = \frac{1}{2}(\omega + \omega_0)\, t$$

Where ω is zero since the flywheel comes to rest and t is in minutes. Simplifying,

$$\theta = \frac{1}{2}\omega_0 t$$

3) What is the magnitude of the tangential component of the linear acceleration of a particle that is located at a distance of 37 cm from the axis of rotation when the flywheel is turning at 86 rev/min?

This problem may be a little confusing because it does not say that it is still related to problem 1. In this problem you are asked to find the tangential component of the acceleration. This is given by

$$a_t = r\alpha$$

But α was found in the first problem in units $\frac{rev}{min^2}$. The radius is given in this problem in cm. Note that the angular speed given has no meaning in this problem. First convert the radius to meters. Then convert α to radians. Finally convert the minutes to seconds. (This last step may be omitted because CAPA knows all units. However, I have not tried this). This can all be done with the following step

$$a_t = r * \frac{1\ m}{100\ cm} * \alpha * \frac{2\pi\ rad}{rev} * \left(\frac{1\ min}{60\ sec}\right)^2$$

or simply

$$a_t = \frac{r}{100}\frac{2\pi\alpha}{3600} = \frac{r\pi\alpha}{180\,000}$$

Remember the sign on α; units, of course, will be in m/s^2. All of the conversions are built into the formula. Thus, you would enter r in cm and α in rev/min^2 (your answer from (1))

4) What is the magnitude of the net linear acceleration of the particle in the above question?

In this problem, you are asked to find the net linear acceleration which means you need to find the other component of the acceleration. Acceleration is a vector and is composed of both tangential and radial components. The radial component of acceleration is

$$a_r = \frac{v^2}{r}$$

But $v = \omega r$, so

$$a_r = \frac{w^2 r^2}{r} = \omega^2 r$$

In question 3, ω is given in revolutions per minute, so you must convert to radians per second. This can be done by the following:

$$a_r = \omega^2 * \left(\frac{2\pi\ rad}{rev}\right)^2 * r * \frac{1\ m}{100\ cm} * \left(\frac{1\ min}{60\ sec}\right)^2$$

or

$$a_r = \omega^2 * 4\pi^2 * \frac{r}{100} * \frac{1}{3600} = \frac{\omega^2\pi^2 r}{90\,000}$$

However, CAPA is asking for the net acceleration. To get this just add a_r and a_t.

$$a_r + a_t = \frac{r}{100}\frac{2\pi\alpha}{3600} = \frac{\omega^2\pi^2 r}{90\,000} + \frac{r\pi\alpha}{180\,000}$$

Again, remember the sign on α (or the sign on your answer to problem #3 if you use that). CAPA is probably picky on the accuracy here, so I'd enter about 7 significant figures in your answer. Remember the units are again in m/s^2. The conversions are all built into the formula. Enter ω in rev/min, r in cm, and α in rev/min^2.

5) M, a solid cylinder ($M = 1.87\ kg$, $R = 0.121\ m$) pivots on a thin, fixed, frictionless bearing. A string wrapped around the cylinder pulls downward with a force F which equals the weight of a 0.710 kg mass, i.e., $F = 6.965\ N$. Calculate the angular acceleration of the cylinder. (Answer in rad/sec^2)

For this question, remember that

$$\tau = rF\sin\theta$$

and can also be related to the moment of inertia by

$$\tau = I\alpha$$

Setting these two equations equal gives

$$rF\sin\theta = I\alpha$$

In the pulley-mass system, the string always acts at right angles to the pulley, so $\theta = 90°$ and $\sin\theta = 1$.

$$rF = I\alpha$$

Solving for α,

$$\frac{rF}{I} = \alpha$$

For a cylinder of solid mass, the moment of inertia is defined to be $\frac{1}{2}Mr^2$. So,

$$\frac{rF}{\frac{1}{2}Mr^2} = \frac{2F}{Mr} = \alpha$$

All quantities are given in SI units, so no conversions should be necessary. The answer should be in rad/sec^2.

6) If instead of the force F an actual mass $m = 0.710\ kg$ is hung from the string, find the angular acceleration of the cylinder.

Start off with a free-body diagram for this problem. All that is necessary is one for the mass. Notice that the forces acting on it are gravity (mg) and the tension (F_t). They act in opposite directions and sum to ma. The mass accelerates downward, however, so a is negative. Thus,

$$\begin{aligned} F_t - mg &= -ma \\ ma &= mg - F_t \end{aligned}$$

So

$$F_t = mg - ma$$

Remember that the linear acceleration is the radius times the angular acceleration, or,

$$a = \alpha R$$

So the equation becomes

$$F_t = mg - m\alpha R$$

For the force on the rotating cylinder,

$$\tau = I\alpha$$

The tension is $\tau = RF_t$ and for a solid cylinder, $I = \frac{1}{2}MR^2$. So,

$$\begin{aligned} RF_t &= \frac{1}{2}MR^2\alpha \\ \alpha &= \frac{2F_t}{MR} \end{aligned}$$

We solved for F_t above, and by plugging this in,

$$\alpha = \frac{2}{MR}(mg - m\alpha R) = \frac{2mg}{MR} - \frac{2m\alpha R}{MR} = \frac{2mg}{MR} - \frac{2m\alpha}{M}$$

Solving for α,

$$\begin{aligned} \alpha + \frac{2m\alpha}{M} &= \frac{2mg}{MR} \\ \alpha\left(1 + \frac{2m}{M}\right) &= \frac{2mg}{MR} \\ \alpha\left(\frac{M+2m}{M}\right) &= \frac{2mg}{MR} \\ \alpha &= \frac{2mg}{MR} * \frac{M}{M+2m} \\ \alpha &= \frac{2mg}{R(M+2m)} \end{aligned}$$

Where m is the mass hung on the string, M is the mass of the cylinder, and R is the radius of the cylinder. The answer will again be in rad/sec^2.

7) A bicycle has wheels with a diameter (DIAMETER, not radius) of 0.600 m. It accelerates uniformly and the rate of rotation of its wheels increases from 189 rpm to 280 rpm in a time of 16.7 s. find the linear acceleration of the bicycle.

Diameter = D = Given
ω_0 = Given
ω = Given
t = Given

Note that t is in seconds and ω is in revolutions per minute. You must convert ω to radians per second with the conversion of

$$\frac{2\pi\ rad}{rev}\frac{1\ min}{60\ sec}$$

CAPA is asking for the linear acceleration of the bicycle, so find the tangential acceleration.

$$a_t = r\alpha$$

Remember that

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

So,

$$a_t = \frac{D}{2} * \frac{\Delta\omega}{\Delta t} = \frac{D}{2} * \frac{(\omega - \omega_0)}{t} * \frac{2\pi\ rad}{rev}\frac{1\ min}{60\ sec}$$

$$a_t = D * \frac{(\omega - \omega_0)}{60t} * \pi$$

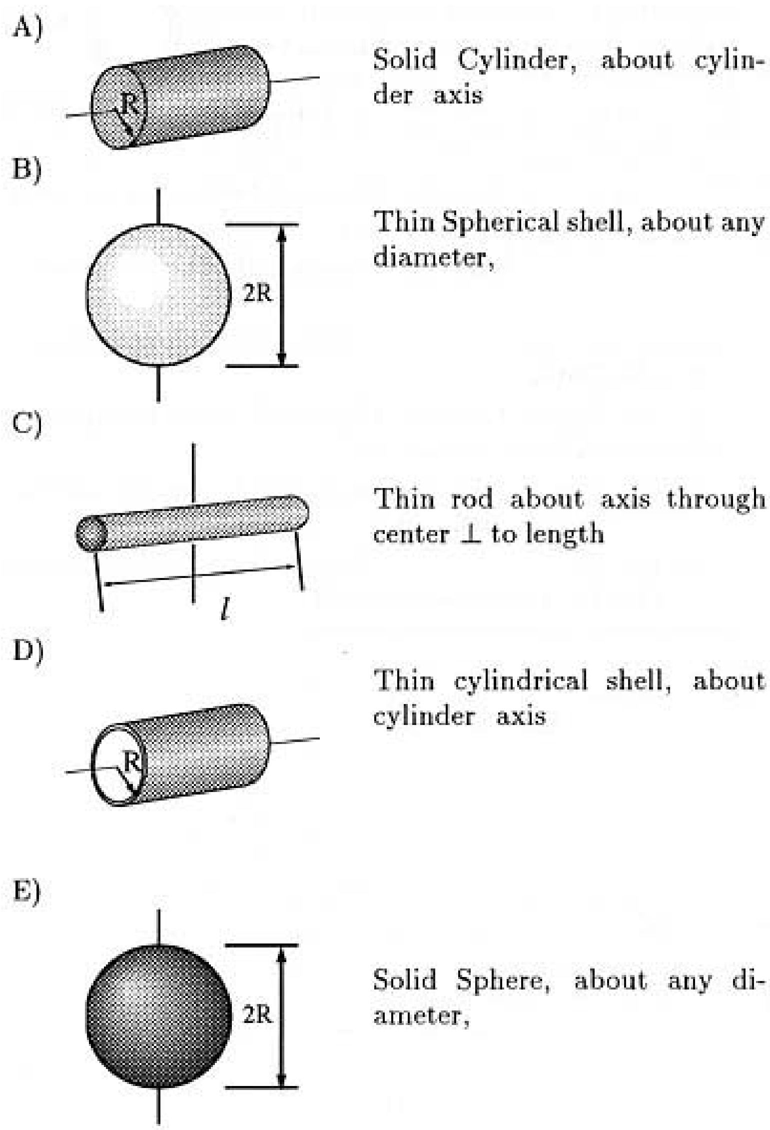
8) Five objects of equal mass are shown below together with the axis about which they are rotating. Select the objects in order of increasing rotational energy. If B has the smallest rotational energy, then A, C, D, and finally E with the largest rotational energy, enter BACDE (Note: If multiple objects have the same rotational energy, then enter them in the order they appear below.

In this problem, several shapes are given and the rotational energies must be calculated. In each case, simply plug in the value of R or l that is given in CAPA to the variables listed below. Remember that if you plug in multiple numbers for the single variable in the equation that all the numbers you plug in must be squared, square rooted, etc.
A) The rotational energy for a solid cylinder is $I = \frac{1}{2}MR^2$
B) The rotational energy for a thin spherical shell is $I = \frac{2}{3}MR^2$
C) The rotational energy for a thin rod about center axis is $I = \frac{1}{12}Ml^2$
D) The rotational energy for a thin cylindrical shell is $I = MR^2$
E) The rotational energy for a solid sphere is $I = \frac{2}{5}MR^2$

On all of these, you need to calculate the rotational kinetic energy. It is given by $KE = \frac{1}{2}I\omega^2$. Note that the term M is virtually worthless since all your answers will all be in terms of it. If you wish, leave M out of the calculations and solve the problem like that.

9) A ball of mass 2.10 kg and radius 0.143 m is released from rest on a plane inclined at an angle $\theta = 41.0°$ with respect to the horizontal. How fast is the ball moving (in m/s) after it has rolled a distance $d = 1.95\ m$? Assume that the ball rolls without slipping, and that its moment of inertia about its center of mass is $1.80 \times 10^{-2}\ kg \cdot m^2$.

m = Given
θ = Given
d = Given
I = Given
r = Given



For this problem, remember that conservation of energy still applies. However, there are just more forms kinetic energy in this problem to keep track of. The basic equation is still

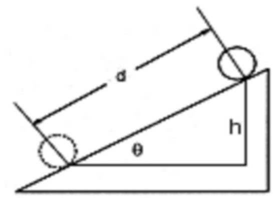
$$KE_i + PE_i = KE_f + PE_f$$

For this problem the ball starts at rest at some initial height and ends up at a lower height where we'll define the potential energy to be zero. That way, the final potential energy is zero.

$$0 + mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega^2 + 0$$

Here, the final kinetic energy is composed of both translational kinetic energy and rotational kinetic energy. We can solve for h this way.

$$h = d\sin\theta$$



Also remember that $v = \omega r$. Thus,

$$mgd\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2}$$

Solving for v,

$$\begin{aligned} 2mgd\sin\theta &= mv^2 + I\frac{v^2}{r^2} \\ 2mgd\sin\theta &= v^2\left(m + \frac{I}{r^2}\right) \\ v &= \sqrt{\frac{2mgd\sin\theta}{m + \frac{I}{r^2}}} \end{aligned}$$

10) The Flintstones and the Rubbles decide to try out the new inclined bowling alley, "Bedslant Bowling". Betty's ball and Fred's ball have the same size, but Fred's ball is hollow. Wilma's ball and Barney's ball are scaled down versions of Betty's ball and Fred's ball respectively. They all place their bowling balls on the same pitch incline and release them from rest at the same time. (Select G-Greater than, L-Less than, E-Equal to,).

For this problem, as covered in the lecture notes, the object with the smallest I will have the greater velocity. So, the smaller the I, the sooner that ball will reach the end. For a solid sphere, $I = \frac{2}{5}MR^2$. A narrow, hallow sphere, $I = \frac{2}{3}MR^2$. Thus, the solid spheres will reach the end before their equal-sized hallow counterparts. To find the final velocity of each object (and consequently the time it takes to reach the end), we can use conservation of energy.

$$\begin{aligned} KE_i + PE_i &= KE_f + PE_f \\ 0 + mgh &= \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega^2 + 0 \qquad \omega = \frac{v}{R} \\ 2mgh &= Mv_f^2 + I\frac{v^2}{R^2} \end{aligned}$$

For the solid sphere,

$$\begin{aligned} 2mgh &= Mv^2 + \frac{2}{5}MR^2\frac{v^2}{R^2} \\ 2gh &= v^2 + \frac{2}{5}v^2 \\ \sqrt{\frac{10}{7}gh} &= v \end{aligned}$$

Note that this does not depend on the radius of the ball, so this is for both large and small solid spheres. Thus, the time it takes the large solid sphere is equal to the time it takes for the smaller solid sphere. For the hallow sphere,

$$\begin{aligned} 2mgh &= Mv^2 + \frac{2}{3}MR^2\frac{v^2}{R^2} \\ 2gh &= v^2 + \frac{2}{3}v^2 \\ \sqrt{\frac{6}{5}gh} &= v \end{aligned}$$

This also does not depend on the radius. Again, the time is the same for both sized hallow spheres. How do the two times compare? They differ only by the constant factor at the beginning of the square root. So, they only differ by how much their constants differ. And

$$\sqrt{\frac{10}{7}} = 1.1952 > \sqrt{\frac{6}{5}} = 1.0954$$

So the solid spheres will take less time (have a greater velocity) than the hallow spheres every time.

11) Two uniform rods are connected to a table by pivots at one end. Rod B is longer than rod A. Both are released simultaneously from an initial angle θ as shown in the figure. Neglect air friction. NOTATION: CM = center of mass; α = angular acceleration; $|a_y|$ = size of downward acceleration. (Give ALL correct answers, i.e., B, AC, BCD...)

QUESTION:
A) The density of the rods affect their rate of fall.
B) Just before landing, the CM of B has a greater speed than the CM of A.
C) α_A and α_B both increase with time.
D) $|a_x|$ of the CM initially equals 0 for both rods.
E) α_A and α_B are dependent on θ.
F) α_A and α_B are the same initially.
G) $|a_y|$ is initially equal for the CM of A and B.
H) Rods A and B hit the table at the same time.

ANSWER:
A)

$$\begin{aligned} \tau &= Fr\sin\theta \\ \tau &= I\alpha \end{aligned}$$

So,

$$I\alpha = Fr\sin\theta$$

Since the force acting on the rod is just mg, and I involves m in some form (for a rod $\frac{1}{3}ml^2$), the m's will cancel.

$$\begin{aligned}
\frac{1}{3}ml^2\alpha &= mgr\sin\theta \\
\frac{1}{3}l^2\alpha &= gr\sin\theta
\end{aligned}$$

So the density will not affect the rate of fall.

B)

This can be found by using conservation of energy. The initial potential energy will equal the final kinetic energy.

$$\begin{aligned}
mgh &= \frac{1}{2}I\omega^2 \\
gh &= \frac{1}{2}\frac{1}{3}l^2\frac{v^2}{\left(\frac{l}{2}\right)^2} \\
gh &= \frac{1}{6}l^2\frac{4v^2}{l^2} \\
gh &= \frac{2}{3}v^2 \\
v &= \sqrt{\frac{3}{2}gh}
\end{aligned}$$

Since the final velocity is proportional to the initial height, the longer rod will hit the ground first.

C)

Although it's hard to see in this picture, the angle θ that the rod is positioned above the horizontal is not the same angle that is involved in the torque calculation from the force of gravity. The angle used in torque is the angle between the rod and vertical (the gravitational force). It is the compliment of θ (that is $90 - \theta$). Although this is not necessary to solve the problem, it is important to see that α for each rod will increase with time. This is because θ will get smaller as the rod approaches the horizontal, but the compliment will grow larger. Since torque is the sine of the compliment, torque will get larger. A larger torque means a larger α.

D)

$|a_x|$ does not initially equal zero because the tension in the rod is pulling the center of mass toward the axis of rotation. This causes an acceleration in the x-direction.

E)

α_A and α_B are dependent on θ. From the equation in part (a),

$$\frac{1}{3}l^2\alpha = gr\sin\theta$$

Thus, $\alpha \propto \theta$.

F)

From the same equation, α_A and α_B are also dependent on the radius from the axis of rotation. Since $a \propto r$, α_A is not equal to α_B.

G)

Since the only force acting in the vertical direction is gravity, $|a_y|$ is the same for both rods.

H)

The rods do not hit the table at the same time. This is a result of (b) where the velocity is dependant on the initial height. Although this is not sufficient in itself to prove this, the shorter rod will actually hit first.

CAPA is looking for the true answers entered in ABC form, so for this problem, the answers would be BCEG.

12) A sledgehammer with a mass of 2.70 kg is connected to a frictionless pivot at the tip of its handle. The distance from the pivot to the center of mass is $r_{cm} = 0.540\ m$, and the moment of inertia about the center of mass is $I_{cm} = 0.0370\ kg \cdot m^2$. If the hammer is released from rest at an angle of $\theta = 48.0°$ such that $H = 0.401\ m$, what is the speed of the center of mass when it passes through horizontal?

$m =$ Given
$r_{cm} =$ Given
$I_{cm} =$ Given
$\theta =$ Given
$H =$ Given

In this problem you are asked to find the final velocity of the center of mass. It is different than the problem worked out in the lecture notes in that for the conservation of energy equation you must also take into account the translational velocity (the center of mass falling vertically). The basic equation still applies

$$\begin{aligned} KE_i + PE_i &= KE_f + PE_f \\ 0 + mgh &= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + 0 \qquad \omega = \frac{v}{r} \end{aligned}$$

$$
\begin{aligned}
2mgh &= mv^2 + I\frac{v^2}{r^2} \\
2mgh &= v^2\left(m + \frac{I}{r^2}\right) \\
v &= \sqrt{\frac{2mgh}{\left(m + \frac{I}{r^2}\right)}}
\end{aligned}
$$